Vectors A level Edexcel Past Paper Answers

01.

Scheme	Marks	AOs
Integrate **v** w.r.t. time	M1	1.1a
$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ \ (+\mathbf{C})$	A1	1.1b
Substitute $t = 4$ and $t = 1$ into their **r**	M1	1.1b
$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
$\sqrt{2^2 + (-30)^2}$	M1	1.1b
$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
	(6)	
		(6 marks)

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute $t = 4$ and $t = 1$ into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

02.3(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5$ m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	

(8 marks)

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Notes: Allow column vectors throughout

(a)

No credit for individual component calculations

M1: Using a complete method to obtain the acceleration. **N.B.** Equation, in **a** only, could be obtained by two integrations

ALTERNATIVE

M1: Use velocity at half-time ($t = 1$) = Average velocity over time period

So at $t = 1$, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$

N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working

A1: Correct **a** vector

M1: Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$

A1*: Correct GIVEN ANSWER obtained correctly

(b)

M1: Using a complete method to obtain the velocity at A e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$ and $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ and their **a**

OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

OR: by integrating their **a**, with addition of $\mathbf{C} = 2\mathbf{i} - 3\mathbf{j}$, and putting $t = 2$

A1: correct vector

M1: Complete method to find equation in t only

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e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components
OR: by integrating $(4\mathbf{i} + 8.8\mathbf{j})$, with addition of a constant, and equating **i** and **j** components.   **N.B.** Must be equating **i** and **j** components of a velocity vector and must be their velocity at A, to give an equation in t only for this M mark   **A1:** 2.5 (s)

03 (a)	$(\mathbf{v} =)\mathbf{C} + (2\mathbf{i} - 3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v} =)(-\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 3\mathbf{j})t$	A1	1.1b
	$\frac{4-3T}{-1+2T} = \frac{-4}{3}$ oe	M1	3.1a
	$T = 8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s} =)\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 \quad (+\mathbf{D})$	M1	3.1a
	$(\mathbf{s} =)(-\mathbf{i} + 4\mathbf{j})t + \frac{1}{2}(2\mathbf{i} - 3\mathbf{j})t^2 \quad (+\mathbf{D})$	A1	1.1b
	$AB = \sqrt{12^2 + 8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2 + 12^2)}$ this is M0A0M0A0.	M1	3.1a
	$= 4\sqrt{13}\,(= 14.422051....)$ (m)	A1cso	1.1b
		(4)	
		(8)	

	Marks	Notes
2a	M1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C} + (2\mathbf{i} - 3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j**) for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in *T* only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get $T = 8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in *T* only, second M0 (ii) Can score M1A1 M1A1 if they get $T = 8$, using trial and error, but if they don't get $T = 8$, can only score max M1A1M0A0

2b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a** = (2**i** – 3**j**) **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i** + 3**j**) for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1cso	Any surd form or 14 or better

04.

i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v}=(t-2t^2)\mathbf{i}+\left(3t-\frac{1}{3}t^3\right)\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$8\mathbf{i}-\frac{28}{3}\mathbf{j}\ (\text{m s}^{-1})$	A1	1.1b
		(3)	
i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t-2t^2+36=0$	A1ft	1.1b
	$t=4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v}=(2t-1)\mathbf{i}+3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2+3^2=5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t=2.5$	A1	1.1b
		(6)	
	(12 marks)		

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Notes: Accept column vectors throughout		
3(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)
	A1	Correct expression
	A1	Accept $8\mathbf{i}-9.3\mathbf{j}$ or better. Isw if speed found.
3(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)
	A1 ft	Correct equation follow through on their **v** but must be a 3 term quadratic
	A1	cao
3(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)
	A1	Correct expression
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2-y^2}$)

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	A1	Correct equation $\sqrt{(2t-1)^2+3^2}=5$
	M1	Solve a 3 term quadratic for t which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working.
	A1	2.5

05. **(a)**	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ with $t=2$: $\mathbf{v}=4\mathbf{i}+2(2\mathbf{i}-3\mathbf{j})$ **OR integration:** $\mathbf{v}=(2\mathbf{i}-3\mathbf{j})t+4\mathbf{i}$,with $t=2$	M1	3.1a
	$\mathbf{v}=8\mathbf{i}-6\mathbf{j}$	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ at $t=3$: $(\mathbf{i}+\mathbf{j})+\left[3\times4\mathbf{i}+\frac{1}{2}\times(2\mathbf{i}-3\mathbf{j})\times3^2\right]$ **OR**: find $\mathbf{v}$ at $t=3$: $4\mathbf{i}+3(2\mathbf{i}-3\mathbf{j})=(10\mathbf{i}-9\mathbf{j})$ then use $\mathbf{r}=\frac{1}{2}(\mathbf{u}+\mathbf{v})t$ $(\mathbf{i}+\mathbf{j})+\left[\frac{1}{2}[4\mathbf{i}+(10\mathbf{i}-9\mathbf{j})]\times3\right]$ or $\mathbf{r}=\mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i}+\mathbf{j})+\left[3\times(10\mathbf{i}-9\mathbf{j})-\frac{1}{2}\times(2\mathbf{i}-3\mathbf{j})\times3^2\right]$ **OR integration:** $\mathbf{r}=(\mathbf{i}+\mathbf{j})+\left[(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2+4t\mathbf{i}\right]$, with $t=3$	M1	3.1a
	$\mathbf{r}=22\mathbf{i}-12.5\mathbf{j}$	A1	2.2a
		(2)	

(4 marks)

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Notes: Accept column vectors throughout		
1a	M1	Complete method to find **v**, using **ruva*t*** or integration (M0 if **i** and/or **j** is missing)
	A1	Apply isw if they also find the speed
1b	M1	Complete method to find the p.v. but this mark can be scored if they omit **(i + j)** i.e. the M1 is for the expression in the square bracket If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$ (M0 if **i** and/or **j** is missing)
	A1	cao

06. **(a)**

$(4\mathbf{i}-\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j})= (4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}$	M1	3.4
Use **ratios** to obtain an equation in λ and μ *only*	M1	2.1
$\frac{(4+\lambda)}{(-1+\mu)}=\frac{3}{1}$ or $\frac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)}=\frac{3}{1}$	A1	1.1b
$\lambda-3\mu+7=0$ * Allow $0=\lambda-3\mu+7$ but nothing else.	A1*	1.1b
	(4)	

(b)	$\lambda = 2 \Rightarrow \mu = 3$; Resultant force $= (6\mathbf{i} + 2\mathbf{j})$ (N)	M1	3.1a
	$(6\mathbf{i} + 2\mathbf{j}) = 4\mathbf{a}$ **OR** $\lvert(6\mathbf{i} + 2\mathbf{j})\rvert = 4a$	M1	1.1b
	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u} = \mathbf{0}$, their $\mathbf{a}$ and $t = 4$: **Or** they may integrate their **a** twice with $\mathbf{u} = \mathbf{0}$ and put $t = 4$: $\mathbf{r} = \frac{1}{2} \times \frac{(6\mathbf{i} + 2\mathbf{j})}{4} 4^2 = (12\mathbf{i} + 4\mathbf{j})$	DM1	2.1
	$\sqrt{12^2 + 4^2}$	M1	1.1b
	ALTERNATIVE 1 for last two M marks: Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: DM1 $s = \frac{1}{2} \times \sqrt{1.5^2 + 0.5^2} \times 4^2$ Use of Pythagoras to find mag of **a** : $a = \sqrt{1.5^2 + 0.5^2}$ M1		
	ALTERNATIVE 2 for last two M marks: Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: DM1 $s = \frac{1}{2} \times \left(\frac{\sqrt{6^2 + 2^2}}{4}\right) \times 4^2$ Use of Pythagoras to find $\lvert(6\mathbf{i} + 2\mathbf{j})\rvert$: $= \sqrt{6^2 + 2^2}$ M1		
	$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1	1.1b
		(5)	

(9 marks)

Notes: Accept column vectors throughout

a	M1	Adding the two forces, **i**'s and **j**'s must be collected (or must be a **single** column vector) seen or implied
	M1	Must be using ratios; Ignore an equation e.g. $(4 + \lambda)\mathbf{i} + (-1 + \mu)\mathbf{j} = 3\mathbf{i} + \mathbf{j}$ if they go on to use ratios.

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	However, if they write $4+\lambda=3$ and $-1+\mu=1$ then $3(-1+\mu)=3$ so $4+\lambda=3(-1+\mu)$ with no use of a constant, it's M0 They may use the acceleration, with a factor of $\frac{1}{4}$ top and bottom, see alternative **Allow one side of the equation to be inverted**
A1	Correct equation
A1*	Given answer correctly obtained. Must see at least one line of working, with the LH fraction 'removed'.

b

M1	Adding $\mathbf{F}_1$ and $\mathbf{F}_2$ to find the resultant force, λ and μ must be substituted **N.B.** M0 if they use $\mu=2$ coming from $-1+\mu=1$ in part (a).
M1	Use of $\mathbf{F}=4\mathbf{a}$ Or $\lvert\mathbf{F}\rvert=4a$, where **F** is <u>their</u> resultant. (including $3\mathbf{i}+\mathbf{j}$) This is an independent mark, so could be earned, for example, if they have subtracted the forces to find the 'resultant' **N.B.** M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$
DM 1	Dependent on previous M mark for **Either:** use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u}=\mathbf{0}$, their **a** and $t=4$ to produce a displacement vector **Or :** integrate twice, with $\mathbf{u}=\mathbf{0}$, their **a** and $t=4$ to produce a displacement Vector **Or:** use of $s=ut+\frac{1}{2}at^2$ with $u=0$, their a and $t=4$ to produce a length
M1	Use of Pythagoras, with square root, to find the magnitude of their displacement vector, **a** or **F** (M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$) depending on which method they have used.
A1	cao

07.

	Scheme	Marks	AOs
a	7**i** – 3**j** seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2+(-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better ($\mathrm{m\ s^{-1}}$)	A1	1.1b
		(3)	
b	$t^2-3t+7=2t^2-3$ OR $\frac{t^2-3t+7}{2t^2-3}=\frac{1}{1}=1$	M1	2.1
	t = 2 only	A1	1.1b
		(2)	
	Differentiate **v** wrt t to give a vector.	M1	3.1a
c	$(2t-3)\mathbf{i}+4t\mathbf{j}$	A1	1.1b
		(2)	
d	$2t-3=0$	M1	3.1a
	t = 1.5	A1	1.1b
		(2)	

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Notes: Allow column vectors throughout.

a	B1	cao
	M1	Use of Pythagoras, including the square root, on a **velocity** vector at $t = 0$
	A1	cao. Must come from a correct **v**.
b	M1	Equating **i** and **j** components of **v** or a ratio of 1:1 to obtain a quadratic in t only. If they use a constant, e.g. $t^2 - 3t + 7 = k$ and $2t^2 - 3 = k$, k must be eliminated to earn this mark. **N.B.** M0 (since wrong working seen) if they write down $\mathbf{i}+\mathbf{j}=(t^2-3t+7)\mathbf{i}+(2t^2-3)\mathbf{j}$ **OR** $\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}t^2-3t+7\\2t^2-3\end{pmatrix}$ **OR** $t^2-3t+7=1$ and $2t^2-3=1$

		and then $t^2-3t+7=2t^2-3$
	A1	$t = 2$
		N.B. Allow M1A1 for a **correct** trial and error method where they obtain $\mathbf{v}=5\mathbf{i}+5\mathbf{j}$ when t = 2 but M0 if they don't get $t = 2$
C	M1	At least one power decreasing by 1 in **each** component in their **v** (M0 if clearly dividing by t) **Both i** and **j** needed in their answer or a column vector Allow recovery if the **i** and **j** disappear and then reappear.
	A1	cao (must be a vector) isw e.g. if they find the magnitude or put $t = 0$ or differentiate again **i**'s and **j**'s do not need to be collected. **N.B.** Allow M1A0 for $2t-3\mathrm{i}+4t\mathrm{j}$
d	M1	$2t - 3 = 0$ or (their **derivative** of the **i**-component of **v**) = 0 **N.B.** M0 if they equate the derivative of both components of **v** to zero.
	A1	cao **N.B.** Correct answer, with no working, can score both marks.