Trigonometry equations and identities As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x = 430.5°, 435.5°$	A1	1.1b
			(5 marks)

Notes

M1: Uses correct identity

A1: Correct three term quadratic

M1: Solves their three term quadratic to give values for $\cos x$ – (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

A1: Two correct answers in the given domain

02.

Question	Scheme	Marks	AOs
a	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2(1-\cos^2\theta)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
		(8 marks)	

Notes

(a)

M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ Note that it cannot just be stated.

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2(1-\cos^2\theta)$ or equivalent

M1: Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$" where y could be $\cos 3x, \cos x$, or even just y. When factorising look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even $\pm$ the correct roots.

B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\frac{2}{3}, -\frac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

03.

Question	Scheme	Marks	AOs
a	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\frac{5}{3}$	M1	2.1
	$x=\text{awrt }121°,301°$	A1 A1	1.1b 1.1b
		(3)	

(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=...$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A,-1<A<1$ or $\cos x=B,-1<B<1$

A1: Either $x=\text{awrt }121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=\text{awrt }121°$ **or** $301°$

Notes On Questions Continue

Alternative proof in part (a):

M1: Multiplies across and form 3TQ in $\cos\theta$ on rhs

$10\sin^2\theta - 7\cos\theta + 2 = (4 - 5\cos\theta)(3 + 2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$

A1: Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$

dM1: Uses $\cos^2\theta = 1 - \sin^2\theta$ on the rhs or $\sin^2\theta = 1 - \cos^2\theta$ on the lhs

Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe

A1*: Shows that $(4 - 5\cos\theta)(3 + 2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

04.

Question	Scheme	Marks	AOs
a	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(\circ)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

05.

Question	Scheme	Marks	AOs
(i)	Uses $\cos^2\theta = 1-\sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \frac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	

(9 marks)

Notes

(i)

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to form a 3TQ in $\sin\theta$

A1: Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

dM1: Solves their 3TQ in $\sin\theta$ to produce one value for θ. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

A1: Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40° = 666.9°$ o.e.

A1: awrt $\alpha = 125°$

06.

Question	Scheme		Marks	AOs
(a)	$\dfrac{1}{\cos\theta}+\tan\theta=\dfrac{1+\sin\theta}{\cos\theta}$ or $\dfrac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$		M1	1.1b
	$=\dfrac{1+\sin\theta}{\cos\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta}=\dfrac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\dfrac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ or $\dfrac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\dfrac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\dfrac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$		dM1	2.1
	$=\dfrac{\cos\theta}{1-\sin\theta}$ *		A1*	1.1b
			(3)	
(b)	$\dfrac{1}{\cos 2x}+\tan 2x=3\cos 2x$ $\Rightarrow 1+\sin 2x=3\cos^2 2x=3\left(1-\sin^2 2x\right)$	$\dfrac{\cos 2x}{1-\sin 2x}=3\cos 2x$ $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$	M1	2.1
	$\Rightarrow 3\sin^2 2x+\sin 2x-2=0$	$\Rightarrow \cos 2x(2-3\sin 2x)=0$	A1	1.1b
	$\sin 2x=\dfrac{2}{3},\ (-1)\Rightarrow 2x=\ldots\Rightarrow x=\ldots$		M1	1.1b
	$x=20.9°,\ 69.1°$		A1 A1	1.1b 1.1b
			(5)	
			(8 marks)	

Notes

(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\dfrac{1+\sin\theta}{\cos\theta}$: Multiplies numerator and denominator by $1-\sin\theta$, uses the difference of two squares and applies $\cos^2\theta=1-\sin^2\theta$
- $\dfrac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$: Uses $\cos^2\theta=1-\sin^2\theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin\theta$ anywhere in the solution and for eg $\sin\theta^2$ instead of $\sin^2\theta$

Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1+\sin\theta}{1+\sin\theta}$ leading to $\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ or

Multiplies by $\frac{\cos\theta}{\cos\theta}$ leading to $\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

dM1: Applies $\cos^2\theta = 1-\sin^2\theta$ and cancels the $\cos\theta$ factor from the numerator and denominator leading to $\frac{1+\sin\theta}{\cos\theta}$ or

Applies $\cos^2\theta = 1-\sin^2\theta$ and uses the difference of two squares leading to $\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by cos $2x$ and applies $\cos^2 2x = 1-\sin^2 2x$ to obtain an equation in sin $2x$ only or alternatively sets $\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ and multiplies by $1-\sin 2x$

A1: Correct equation or equivalent. The = 0 may be implied by their later work (Condone notational slips in their working)

M1: Solves for sin $2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x. The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in sin$2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

07.

Question	Scheme	Marks	AOs
a	States or uses $\tan x = \frac{\sin x}{\cos x}$	B1	1.2
	$4\sin x = 5\cos^2 x \Rightarrow 4\sin x = 5(1-\sin^2 x)$	M1	1.1b
	$5\sin^2 x + 4\sin x - 5 = 0$ *	A1*	2.1
		(3)	
(b)	Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow \sin x = ...$	M1	1.1b
	$\sin x = \frac{-2\pm\sqrt{29}}{5}$ $(\sin x = \text{awrt } 0.677)$	A1	1.1b
	Takes $\sin^{-1}$ leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{°\}$ and $x = \text{awrt } 137.4\{°\}$ only	A1	1.1b
		(4)	
(c)	$15\times"2"=30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3\times5\times$ their number in range 0 to 360° or 'in words" e.g., stating 3 ×"2" values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	
		(9 marks)	

Notes:

(a) **Allow use of e.g. θ but the final mark requires the equation to be in terms of x**

B1: States or uses $\tan x = \frac{\sin x}{\cos x}$ e.g., $4\tan x = 5\cos x \Rightarrow 4\frac{\sin x}{\cos x} = 5\cos x$ Allow e.g. $\tan x = \frac{\sin\theta}{\cos\theta}$

M1: Multiplies by $\cos x$ and uses $\cos^2 x = 1-\sin^2 x$ to set up a quadratic equation in just $\sin x$. Condone mixed arguments here.

A1*: Proceeds to $5\sin^2 x + 4\sin x - 5 = 0$ with correct notation and algebra, showing all key steps. The $= 0$ must be present in the final answer line. Condone a single slip in notation, e.g., $\sin x^2$ or $\sin\theta$ seen once.

(b)

M1: Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow$ **$\sin x = ...$** using the usual rules. $\sin x =$ may be implied later. Allow solution(s) from a calculator but one must be correct (0.6 or 0.7 or –1.4 or –1.5)

A1: Achieves $\sin x = \frac{-4\pm\sqrt{116}}{10}$ $(\sin x = \text{awrt } 0.677)$ $\sin x =$ may be implied later.

dM1: Finds one value of x in the range 0 to 360° from their $\sin x =$
May be scored for working in radians. If using $\sin x = 0.677$ they should have awrt 0.744 or awrt 2.40
If they have made a slip in solving the quadratic, e.g., by the formula, then their values will need checking both in degrees and radians to see if this mark can be implied.

A1:	$x =$ awrt $42.6\{°\}$ and $x =$ awrt $137.4\{°\}$ only. Ignore any values outside of 0 to 360° isw if they round their values to e.g., 3sf after stating acceptable answers. There must be some evidence that the quadratic has been solved.
(c)	
B1ft:	Follow through on 15 multiplied by the number of solutions in (b) in the range 0 to 360° If working in radians in (b), they must state 30 (solutions).
B1ft:	Explains either mathematically **or** in words. See scheme. Note that you might see arguments expanding the range from 1800 to 5400 to account for the stretch parallel to the x axis. $\frac{5400}{360} = 15$ and $15 \times 2 = 30$ which is also acceptable.
Note:	If candidates list 30 values and conclude that there are 30 solutions, score B1ftB1ft There is no need to check their 30 values are correct, but there must be 30.