

International GCSE Further Pure Mathematics – Paper 1 mark scheme

Paper 1				
Question	Working	Answer	Mark	AO
1 (a)				
(b)				
			B1 B1	1
			B1 B1	1
			(4)	
				One correct line Both correct lines Correct line $x = 2$ drawn Correct region shaded in or out.

Question	Working	Answer	Mark	AO	Notes
2 (a)	$\pi = 6\theta \Rightarrow \theta = \frac{\pi}{6} \Rightarrow \angle AOB = \frac{5\pi}{6} = (150^\circ)$ $AB = \sqrt{10^2 + 6^2 - 2 \times 10 \times 6 \times \cos\left(\frac{5\pi}{6}\right)} = 15.4894... = 15.5 \text{ cm}$	15.5 (cm)	B1	1	Accept working in degrees
(b)	$\text{Area} = \frac{1}{2} \times 10 \times 6 \times \sin \frac{5\pi}{6} + \frac{\pi}{6} \times \frac{6^2}{2} = 24.424...$	24.4 (cm ²)	M1M1A1	2	$\angle AOB = 150^\circ$
	ALTERNATIVE $\text{Area} = \frac{1}{2} \times 10 \times 6 \times \sin \frac{5\pi}{6} + \frac{1}{2} \times \pi \times 6 = 24.424...$		M1M1A1 (6)		
3	$2 \cos(2\theta + 30) + \frac{\sin(2\theta + 30)}{\cos(2\theta + 30)} = 0 \Rightarrow 2 \cos^2(2\theta + 30) + \sin(2\theta + 30) = 0$ $\Rightarrow 2 - 2 \sin^2(2\theta + 30) + \sin(2\theta + 30) = 0$ $\sin(2\theta + 30) = \frac{1 \pm \sqrt{1 - 4 \times 2 \times (-2)}}{2 \times 2} = 1.2807..., -0.7807...$ $2\theta + 30 = -51.33167..., 231.33167, 308.66833$ $\theta = 100.7, 139.3$	$\theta = 100.7, 139.3$	M1 M1A1 M1 A1A1 (6)	2 3	Solves 3 TQ Finds one angle from their 3TQ

Question	Working	Answer	Mark	AO	Notes
4					
(a)	$v = 0$ so $4t^2 - 19t + 12 = 0 \Rightarrow (4t - 3)(t - 4) = 0 \Rightarrow t = \frac{3}{4}, 4$	$t = \frac{3}{4}, 4$	M1A1	1	
(b)	$s = \int 4t^2 - 19t + 2 dt = \frac{4t^3}{3} - \frac{19t^2}{2} + 12t + c$ when $t = 0, s = -4 \Rightarrow c = -4$ When $t = 6, s = \frac{4 \times 6^3}{3} - \frac{19 \times 6^2}{2} + 12 \times 6 - 4 = 14$	14	M1M1A1	2	
(c)	$a = \frac{dv}{dt} = 8t - 19 \Rightarrow 8t - 19 = 0 \Rightarrow t = \frac{19}{8}$	$t = \frac{19}{8}$	A1 M1M1A1 (9)	3	
5	$2x + y = 13 \Rightarrow y = 13 - 2x$		B1	1	
(a)	$S = 4x^2 + (13 - 2x)^2 = 4x^2 + 169 - 52x + 4x^2 = 8x^2 - 52x + 169$	$8x^2 - 52x + 169$	M1A1		
(b)	$\frac{dS}{dx} = 16x - 52, \frac{dS}{dx} = 0 \Rightarrow 16x - 52 = 0 \Rightarrow x = \frac{13}{4}$ $\frac{d^2S}{dx^2} = 16, 16 > 0,$	$x = \frac{13}{4}$ Hence minimum	M1M1A1 B1	2,3	
(c)	$S = 8 \times \left(\frac{13}{4}\right)^2 - 52 \times \frac{13}{4} + 169 = \frac{169}{2} = 84.5$	$\frac{169}{2} = 84.5$	M1A1 (9)	3	

Question	Working	Answer	Mark	AO	Notes														
6	$\frac{dy}{dx} = e^x(x^2 - 3x) + e^x(2x - 3) \Rightarrow e^x(2x - 3) = \frac{dy}{dx} - y$ $\frac{d^2y}{dx^2} = e^x(x^2 - 3x) + e^x(2x - 3) + e^x(2x - 3) + 2e^x = y + 2\left(\frac{dy}{dx} - y\right) + 2e^x$ $2e^x = \frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx} - y\right) - y \Rightarrow 2e^x = y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} *$		M1M1A1 M1A1 M1M1A1 (8)	4 4 (8)															
7	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>3</td><td>3.83</td><td>5</td><td>6.66</td><td>9</td><td>12.31</td></tr></table> (a) All points plotted within an accuracy of half of a square. A smooth curve drawn through their points (b) (c) $\log_2(4x - 6)^2 - x = 2 \Rightarrow 2\log_2(4x - 6) = x + 2 \Rightarrow \log_2(4x - 6) = \frac{x}{2} + 1$ $\Rightarrow 4x - 6 = 2^{\left(\frac{x}{2} + 1\right)} \Rightarrow 4x - 5 = 2^{\left(\frac{x}{2} + 1\right)} + 1$ Line $y = 4x - 5$ drawn on graph $\Rightarrow$ so $x = 2.8(36)$	x	0	1	2	3	4	5	y	3	3.83	5	6.66	9	12.31	$x = 2.8$ (8)	B1B1 B1B1 M1M1 M1A1 (8)	1 1 2 (8)	
x	0	1	2	3	4	5													
y	3	3.83	5	6.66	9	12.31													

Question	Working	Answer	Mark	AO	Notes
8					
(a)	$a = S_1 = 2 \times 1 \times (1 + 3) = 8$	$a = 8$	B1	1	
(b)	$S_2 = 2 \times 2 \times (2 + 3) = 20$ $S_2 = a + T_2 \Rightarrow T_2 = S_2 - a = 20 - 8 = 12$ $d = 12 - 8 = 4$	$d = 4$	M1A1	1	
(c)	Uses given formula for $S_n = 2n(n+3)$ and formula for nth term $T_n = a + (n - 1)d$ $6[2(n - 4)(n - 4 + 3)] = 7[8 + (n + 3 - 1)4]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n - 8)(3n + 2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3} \right)$ ALT Using formula $S_n = \frac{n}{2}(2a + (n - 1)d)$ $6\left[\frac{(n - 4)}{2}(2 \times 8 + ((n - 4) - 1)4)\right] = 7[8 + (n + 3 - 1)4]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n - 8)(3n + 2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3} \right)$	$n = 8$	M1M1 M1A1	2, 3	
			M1A1		
			M1M1 M1A1	2, 3	
			M1A1		
			(9)		

Question	Working	Answer	Mark	AO	Notes
9					
(a)	$\alpha + \beta = -\frac{7}{3}$ $\alpha\beta = -2 = -\frac{6}{3}$ so $a = 3$, $b = 7$ and $c = -6$ Hence quadratic equation $\Rightarrow 3x^2 + 7x - 6 = 0$ oe with integer coefficients		B1B1 M1A1	1	
(b)	$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = (\alpha + \beta)^2 - 4\alpha\beta = \left(-\frac{7}{3}\right)^2 - 4 \times -2 = \frac{121}{9}$ $\alpha > \beta$ so $\alpha - \beta = \frac{11}{3}$ *		M1A1	3	
(c)	Sum $\frac{\alpha + \beta}{\alpha} + \frac{\alpha - \beta}{\beta} = \frac{\beta(\alpha + \beta) + \alpha(\alpha - \beta)}{\alpha\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\left(-\frac{7}{3}\right)^2 - 2 \times -2}{-2} = -\frac{85}{18}$ Product $\frac{(\alpha + \beta)}{\alpha} \times \frac{(\alpha - \beta)}{\beta} = \frac{\left(-\frac{7}{3}\right) \times \left(\frac{11}{3}\right)}{-2} = \frac{77}{18}$ Equation $18y^2 + 85y + 77 = 0$ oe with integer coefficients		M1M1A1 M1A1 M1A1 (13)	3	

Question	Working	Answer	Mark	AO	Notes
10	Let M be the midpoint of diagonals AC or DB				
(a)	$AC = \sqrt{8^2 + 8^2} = 8\sqrt{2} \Rightarrow AM = 4\sqrt{2}$ $h = \sqrt{12^2 - (4\sqrt{2})^2} = \sqrt{112} = (4\sqrt{7})$	$4\sqrt{7}$	M1M1A1	1	
(b)	$\tan^{-1}\left(\frac{4\sqrt{7}}{4\sqrt{2}}\right) = 61.87449... \approx 61.9^\circ$	61.9°	M1A1	1	Or any equivalent trigonometry
(c)	Let N be the midpoint of AB				
	Angle the plane AOB makes with horiz = $\tan^{-1}\left(\frac{4\sqrt{7}}{4}\right) = 69.295... \approx 69.3^\circ$	69.3°	M1A1	3	Or any equivalent trigonometry
(d)	By using the symmetrical properties of the pyramid Let S be the perpendicular from P to diagonal AC Let R be the perpendicular from S to side BC In triangle $PSR \rightarrow PR = \sqrt{(2\sqrt{17})^2 - 2^2} = 8$ In triangle $PRQ \rightarrow PQ = \sqrt{8^2 + 4^2} = 4\sqrt{5}$	$4\sqrt{5}$	M1A1 M1A1	3	Or any equivalent system of right angle triangles
(e)	Length $AQ = \sqrt{8^2 + 6^2} = 10$ Angle of $PQA = \cos^{-1}\left(\frac{10^2 + (4\sqrt{5})^2 - 6^2}{2 \times 10 \times 4\sqrt{5}}\right) = 36.39124... \approx 36.4^\circ$	36.4°	M1 M1A1A1 (15)	3	

Question	Working	Answer	Mark	AO	Notes
11	Mark parts (i) and (ii) together			1, 3	
(a)	$y = \int 6x^2 - 26x + 12 \, dx = \left[\frac{6x^3}{3} - \frac{26x^2}{2} + 12x + c \right]$ At the point $(-1, 0)$ $0 = 2(-1)^3 - 13(-1)^2 + 12(-1) + C \Rightarrow C = 27$		M1		
			M1A1		
	$(2x^3 - 13x^2 + 12x + 27) \div (x + 1) = 2x^2 - 15x + 27 = (2x - 9)(x - 3)$ $\Rightarrow a = \frac{9}{2}, b = 3$	$a = \frac{9}{2}, b = 3$	M1M1A1 B1B1		
(b)	$\text{Area} = \int_0^3 2x^3 - 13x^2 + 12x + 27 \, dx + \left \int_3^9 2x^3 - 13x^2 + 12x + 27 \, dx \right =$ $\left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_0^3 + \left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_3^9 = \frac{2043}{32}$ So Area = $\frac{2043}{32}$	$\text{Area} = \frac{2043}{32}$	M1M1 A1M1A1	3	
			(13)		
		Total	100		