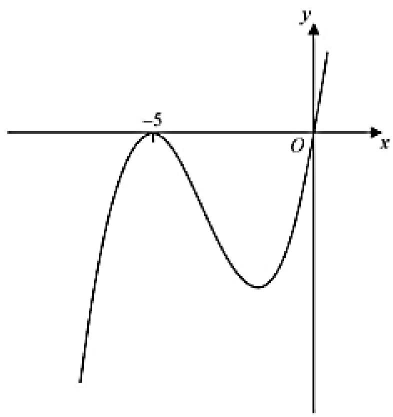
Sketching graphs As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
a	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes

(a) M1: Takes out factor x

A1: Correct factorisation - allow $x(x+5)(x+5)$

(b) M1: Correct shape

A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c) M1: May be implied by one of the correct answers for a or by a statement

A1ft: ft from their cubic as long as it meets the x-axis only twice.

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02.

Question	Scheme	Marks	AOs
a	$(g(-2)) = 4\times-8-12\times4-15\times-2+50$	M1	1.1b
	$g(-2)=0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$	M1 A1	1.1b 1.1b
	$=(x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x=2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x=-1,\ x=1.25$	B1ft	2.2a
		(3)	
		(9 marks)	

Notes

(a)

M1: Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3-12\times(-2)^2-15\times(-2)+50$ embedded

Or $-32-48+30+50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2)=0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$g(-2)=0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3-12x^2-15x+50=(x+2)(4x^2 \text{..........} \pm 25)$

If algebraic / long division is used expect to see

$$\begin{array}{r} 4x^2 \pm 20x \\ x+2\overline{)\,4x^3-12x^2-15x+50} \end{array}$$

A1: Correct quadratic factor is $(4x^2-20x+25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2-20x+25)$ usual rule $(ax+b)(cx+d), ac=\pm4, bd=\pm25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $g(x)=(x+2)(x-2.5)^2=(x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x=2.5$ Follow through on their $g(x)=(x+2)(ax+b)^2$ only where $ab<0$ (that is a positive root). Condone $x<-2$ See SC below for $g(x)=(x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

....

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

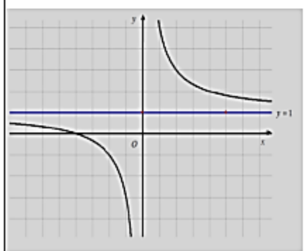
In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

....

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either a or b	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

03.

Question	Scheme	Marks	AOs
a	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x) \Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
		(8 marks)	

Notes

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow "1-\frac{1}{2}k^2"=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct

Notes On Questions Continue

A1: $k = \pm\sqrt{2}$ and following correct a, b and c if stated

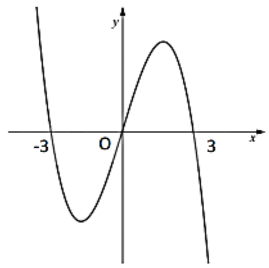
A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $= -2 \Rightarrow -\frac{k^2}{x^2} = -2 \Rightarrow x = (\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2 - 4x + k^2 = 0$

A1: $2k^2 = (\pm)2\sqrt{2}k$ oe

A1: $k = \pm\sqrt{2}$

04.

Question	Scheme		Marks	AOs
a	$9x - x^3 = x(9 - x^2)$		M1	1.1b
	$9x - x^3 = x(3-x)(3+x)$ oe		A1	1.1b
			(2)	
(b)		A cubic with correct orientation	B1	1.1b
		Passes though origin, (3, 0) and (−3, 0)	B1	1.1b
			(2)	
(c)	$y = 9x - x^3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = ...$		M1	3.1a
	$y = (\pm)6\sqrt{3}$		A1	1.1b
	$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$ oe		A1ft	2.5
			(3)	
			(7 marks)	

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x(\pm x \pm 3)(\pm x \pm 3)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x(3-x)(3+x)$ on its own scores M1A1.

Allow eg $-x(x-3)(x+3)$, $x(x-3)(-x-3)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x+0)$.

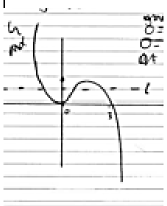
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

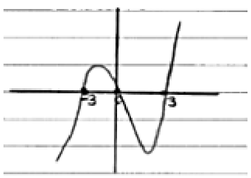
B1: Passes **through** each of the origin, (3, 0) and (−3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and -3 on the axes. Condone x and y to be the wrong way round eg $(0, -3)$ for $(-3, 0)$ as long as it is on the correct axis but do not allow (−3, 0) to be labelled as (3, 0).

Examples

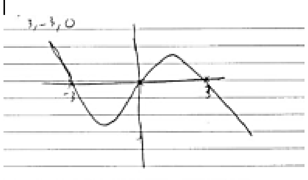
B1B0

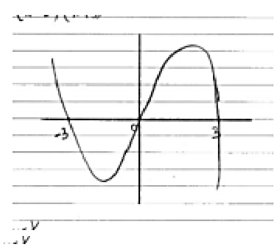


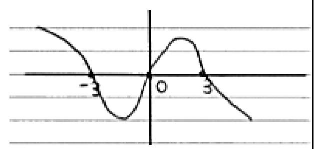
B0B1



B1B1







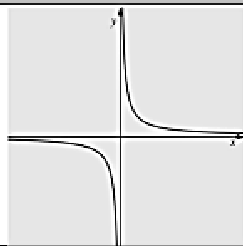
(c) ***Be aware the value of y can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{dy}{dx}=0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.
Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.
Condone $\{"-6\sqrt{3}" < k < "6\sqrt{3}"\}$ or $\{"-6\sqrt{3}" < k\} \cap \{k < "6\sqrt{3}"\}$ but not $\{"-6\sqrt{3}" < k\} \cup \{k < "6\sqrt{3}"\}$
Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.
Must be in terms of k

05.

Question	Scheme	Marks	AOs
a	 Shape in quadrant 1 **or** 3	M1	1.1b
	Shape and Position	A1	1.1b
		(2)	
(b)	Deduces that $x < 0$	B1	2.2a
	Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$	M1	1.1b
	$x < 0$ or $x \geqslant 8$	A1 cso	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For the correct shape in quadrant 1 **or** 3. Do not be concerned about position but it must not cross either axis. Ignore incorrect asymptotes for this mark.

A1: Correct shape and position. There should be no curve in either quadrant 2 or quadrant 4. The curve must not clearly bend back on itself but condone slips of the pen.

(b)

B1: Deduces that $x < 0$ but condone $x \leqslant 0$ for this mark.

M1: Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$ where the … means any equality or inequality.

A1: cso $x < 0$ or $x \geqslant 8$ (Both required)

Set notation may be seen $\{x : x < 0\} \cup \{x : x \geqslant 8\}$ o.e. $x \in (-\infty, 0) \cup [8, \infty)$

Accept $x < 0, x \geqslant 8$ but not $x < 0$ and $x \geqslant 8$

Must not be combined incorrectly, e.g., $8 \leqslant x < 0$ **or** $\{x : x < 0\} \cap \{x : x \geqslant 8\}$