Quadratics As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
a	States or uses $f(+3)=0$	M1	1.1b
	$4(3)^3-12(3)^2+2(3)-6=108-108+6-6=0$ and so $(x-3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so $4x^3-12x^2+2x-6=(x-3)(4x^2+...)$	M1	2.1
	$4x^3-12x^2+2x-6=(x-3)(4x^2+2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2+2)=0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2+2>0$ for all x) So $x=3$ is the only real root of $f(x)=0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes

(a) M1: States or uses $f(+3)=0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b) M1: Needs to have $(x-3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x-3)(2x^2+1)$

M1: Considers **their** quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: a correct explanation.

02.

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes 3 = 0 (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

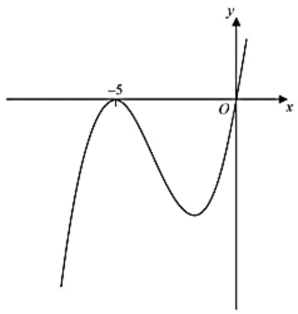
B1 : Explains why $k = 0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

03.

Question	Scheme	Marks	AOs
a	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes

(a) M1: Takes out factor x
A1: Correct factorisation - allow $x(x+5)(x+5)$

(b) M1: Correct shape
A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c) M1: May be implied by one of the correct answers for a or by a statement
A1ft: ft from their cubic as long as it meets the x-axis only twice.
A1ft : ft from their cubic as long as it meets the x-axis only twice.

04.

Question	Scheme	Marks	AOs
a	Attempts $P = 100 - 6.25(15-9)^2$	M1	3.4
	$= -125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x-9)^2 < 3.2$ or $P = 80 \Rightarrow (x-9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x = 15$ into $P = 100 - 6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the $x = 15$. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P = -125$ or states that $P < 0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P = -125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. (They will lose marks later in the question). An explanation that it is not sensible as "the profit cannot be negative" , "the profit is negative" or "the company will not make any money" , "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets $P = 0$ and finds $x = 5, 13$ A1: States $15 > 13$ and states makes a loss

(b)

M1: Uses $P ... 80$ where ... is any inequality or "=" in $P = 100 - 6.25(x-9)^2$ and proceeds to $(x-9)^2 ... k$ where $k > 0$ and ... is any inequality or "="

Eg. Condone $P < 80$ in $P = 100 - 6.25(x-9)^2 \Rightarrow (x-9)^2 < k$ where $k > 0$ If the candidate attempts to multiply out then allow when they achieve a form $ax^2 + bx + c = 0$

dM1: Award for solving to find the two positive values for x. Allow decimal answers

FYI correct answers are $\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$ Accept $\Rightarrow x = 9 \pm \sqrt{3.2}$

Condone incorrect inequality work $100 - 6.25(x-9)^2 > 80 \Rightarrow (x-9)^2 > 3.2 \Rightarrow x > 9 \pm \sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9 - \sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

Trial and improvement or just answers of £7.22 or £7.21(with no working) then please send to review.

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound.

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once, withhold the final mark. Eg correct values in (c) would be scored B1 B0.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

05.

Question	Scheme	Marks	AOs
a	$(g(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
		(9 marks)	

Notes

(a)

M1: Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3 - 12\times(-2)^2 - 15\times(-2) + 50$ embedded

Or $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2) = 0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$g(-2) = 0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \ldots\ldots\ldots \pm 25)$

If algebraic / long division is used expect to see $x+2\,\overline{)\,4x^3 - 12x^2 - 15x + 50}$ with quotient $4x^2 \pm 20x$

A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d), ac = \pm 4, bd = \pm 25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $g(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x = 2.5$ Follow through on their $g(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $g(x) = (x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

....

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

....

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either a or b	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

06.

Question	Scheme		Marks	AOs
(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2 + 9)(b^2 - 2) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
			(8 marks)	

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

Eg. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $(b^2 + m)(b^2 + n) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b=\sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Notes On Questions Contiune

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

07.

Question	Scheme	Marks	AOs
a	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \{1200-3\times(4-20)^2\}-\{1200-3\times(3-20)^2\}$

A1: 93 tonnes or 93 t
(d)
For one mark
Shows an appreciation of the model
- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks
States the limitation on n and explains fully. (Total mass, not mass must be used)
- States that $n \leqslant 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that n cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \leqslant 20$ as the maximum total amount of tin mined is reached at 20 years

08.

Question	Scheme	Marks	AOs
(i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\frac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\frac{\sqrt{18}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\frac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\frac{3}{2} \Rightarrow x=...$	dM1	1.1b
	$x=\frac{5}{12}$	A1	1.1b
		(3)	
		(6 marks)	

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\frac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^c$ or $4^{dx+e}=4^f$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\frac{1}{2\sqrt{2}} \Rightarrow 2(3x-2)=\log_2\frac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\frac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2} \times \frac{1}{2}$ condoning slips as per main scheme

or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1 + \frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

09.

Question	Scheme	Marks	AOs
a	$g(5)=2\times5^3+5^2-41\times5-70=...$	M1	1.1a
	$g(5)=0\Rightarrow(x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3+x^2-41x-70=(x-5)(2x^2...x\pm14)$	M1	1.1b
	$=(x-5)(2x^2+11x+14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x))=(x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3+x^2-41x-70\,dx=\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3}-\frac{190}{3}$	M1	2.2a
	Area = $571\frac{2}{3}$	A1	2.1
		(4)	
		(10 marks)	

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0
Look for evidence of embedded values or two correct terms of $g(5)=250+25-205-70=...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$g(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$

$g(5)=0\Rightarrow(x-5)$ is a factor ✓

Do not allow if candidate states

$f(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**

$g(x)=0\Rightarrow(x-5)$ is a factor **(It is not g(x) and there is no conclusion)**

This may be seen in a preamble before finding $g(5)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2+11x+14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2+11x+14)$

A1: $(\mathrm{g}(x)=)\ (x-5)(2x+7)(x+2)$ or $(\mathrm{g}(x)=)\ (x-5)(x+3.5)(2x+4)$
It is for the product of factors and not just a statement of the three factors
Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \to x^{n+1}$ for any of the terms in x for $\mathrm{g}(x)$ so
$2x^3 \to ...x^4,\ x^2 \to ...x^3,\ -41x \to ...x^2,\ -70 \to ...x$

A1: $\int 2x^3+x^2-41x-70\,\mathrm{d}x = \frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to area = $571\frac{2}{3}$ oe

So allow $\int_{-2}^{5} 2x^3+x^2-41x-70\,\mathrm{d}x = \left[\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x\right]_{-2}^{5} = -\frac{1715}{3} \Rightarrow$ area $=\frac{1715}{3}$
for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x$ then withhold the final mark if they just write a positive value to this integral since $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x = -\frac{1715}{3}$

Note $\int_{-2}^{5} 2x^3+x^2-41x-70\,\mathrm{d}x \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

10.

Question	Scheme	Marks	AOs
a	$3x^3-17x^2-6x=0 \Rightarrow x(3x^2-17x-6)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\frac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution …0 to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
			(6 marks)

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x=0,-\frac{1}{3},6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2=n$ where n is any solution …0 to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2=0 \Rightarrow y=2$

A1ft: Two of $2, 2\pm\sqrt{6}$ but follow through on $(y-2)^2=n \Rightarrow y=2\pm\sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2\pm\sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

11.

Question	Scheme	Marks	AOs
a	$f(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + ...$	M1	1.1b
	$= -3(x-2)^2 + ...$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8\,dx = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 (-3x^2 + 12x + 8)\,dx$	M1	3.1a
	$R = 40 - [-2^3 + 24 + 16]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
		(10 marks)	
Alt(c)	$\int 3x^2 - 12x + 12\,dx = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12\,dx$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + ...$
Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + ...$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $(-b, c)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{"2"} f(x) \, dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

12.

Question	Scheme	Marks	AOs
a	$f(-3)=2(-3)^3+5(-3)^2+2(-3)+15$ $=-54+45-6+15$	M1	1.1b
	$f(-3)=0 \Rightarrow (x+3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a=2,\ b=-1,\ c=5$	M1	1.1b
	All of: $a=2,\ b=-1,\ c=5$	A1	1.1b
		(2)	
(c)	$b^2-4ac=(-1)^2-4(2)(5)$	M1	2.1
	$b^2-4ac=-39$ which is <0 so the quadratic has no real roots so $f(x)=0$ has only 1 real root	A1	2.4
		(2)	
(d)	$(x=)\ 2$	B1	2.2a
		(1)	
			(7 marks)

Notes

(a)

M1: Attempts f(−3). Attempted division by $(x+3)$ or $f(3)$ is M0
Look for evidence of embedded values or two correct terms of
$f(-3)=-54+45-6+15=...$

A1: Achieves and states f(−3) = 0, and makes a suitable conclusion. Sight of f(x)=0 when $x=-3$ is also acceptable.
It must follow M1. Accept, for example, $f(-3)=0 \Rightarrow (x+3)$ is a factor

This may be seen in a preamble before finding $f(-3)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Correct method implied by values for at least 2 correct constants. Allow embedded in their $f(x)$ or within their working if they use algebraic division/other methods which may be seen in part (a) and used in part (b).

A1: All values correct. Allow embedded in their $f(x)$ or seen as the quotient from algebraic division. Isw incorrectly stated values of a b and c following a correct quadratic expression seen.

$$\begin{array}{r} 2x^2 - x + 5 \\ x+3 \overline{)2x^3 + 5x^2 + 2x + 15} \\ 2x^3 + 6x^2 \quad\quad\quad\quad \\ -x^2 + 2x \quad\quad \\ -x^2 - 3x \quad\quad \\ 5x + 15 \\ 5x + 15 \\ 0 \end{array}$$

scores M1A1

(c)

M1: Either:

- considers the discriminant using their a, b and c (does not need to be evaluated) $(b^2 - 4ac =)\ (-1)^2 - 4(2)(5)$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2). Discriminant $= -39$ is sufficient for M1
- attempts to complete the square so score for $2\left(x \pm \frac{1}{4}\right)^2 + \ldots$
- attempts to find the roots of the quadratic using the formula. The values embedded in the formula score this mark. $\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 5}}{2 \times 2}$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2)
- Sketches a graph of the quadratic. It must be a U shaped quadratic which does not cross the x-axis.

A1: Provides a correct explanation from correct working. They must

- Have a correct calculation
- Explanation that the quadratic has no (real) roots
- Minimal conclusion stating that $f(x) = 0$ has only one root

eg $b^2 - 4ac = -39 < 0$ so only one root is M1A0 (needs to explain the quadratic has no real roots)

eg $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8} > 0$ **so no real roots** (for the quadratic) **so** (f(x) has) **only one** (real) **root** is M1A1

The value of the discriminant, completed square form $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8}$ or roots of the quadratic $\left(= \frac{1 \pm \sqrt{39}i}{4}\right)$ must be correct.

If they sketch the quadratic graph it must be a U shaped quadratic which crosses the y-axis at 5 and has a minimum in the 1st quadrant. They must explain that the graph does not cross the x-axis so no real roots for the quadratic so only one root for $f(x) = 0$.

(d)

B1: 2 condone (2, 0)

13.

Question	Scheme	Marks	AOs
	Let $u=\sqrt{x}$ $\quad 6x+7\sqrt{x}-20=0 \Rightarrow 6u^2+7u-20=0$ $\Rightarrow (3u-4)(2u+5)\{=0\}$	M1A1	1.1b 1.1b
	Attempts $\sqrt{x}=\text{"}\frac{4}{3}\text{"},\text{"}-\frac{5}{2}\text{"} \Rightarrow x=...$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
		(4 marks)	
Alt 1	$6x+7\sqrt{x}-20=0 \Rightarrow 7\sqrt{x}=20-6x \Rightarrow 49x=(20-6x)^2$ $\Rightarrow 49x=400-240x+36x^2$	M1	1.1b
	$36x^2-289x+400\{=0\}$	A1	1.1b
	$(9x-16)(4x-25)=0$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
Alt 2	$6x+7\sqrt{x}-20=0 \Rightarrow (3\sqrt{x}-4)(2\sqrt{x}+5)=0$	M1 A1	1.1b 1.1b
	Attempts $\sqrt{x}=\text{"}\frac{4}{3}\text{"},\text{"}-\frac{5}{2}\text{"} \Rightarrow x=...$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	

Notes:

M1: Attempts a valid method that enables the problem to be solved. See General Principles for Pure Mathematics Marking at the front of the mark scheme for guidance. Score for either letting $u=\sqrt{x}$ and attempting to factorise to $(au\pm c)(bu\pm d)$ with $ab=6, cd=20$

or making $7\sqrt{x}$ the subject and attempting to square both sides.

or attempting to factorise to $(a\sqrt{x}\pm c)(b\sqrt{x}\pm d)$ with $ab=6, cd=20$

or by quadratic formula or completing the square following usual rules.

A1: $(3u-4)(2u+5)\{=0\}$ **or** $36x^2-289x+400\{=0\}$ **or** $(3\sqrt{x}-4)(2\sqrt{x}+5)\{=0\}$

If they use the formula, it must be correct e.g., $u\{\text{or}\sqrt{x}\}=\frac{-7\pm\sqrt{7^2-4(6)(-20)}}{12}$ followed by $u\{\text{or}\sqrt{x}\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\{\text{or}\sqrt{x}\}=-\frac{5}{2}$ or not.

	If they complete the square, they must have $\left(u+\frac{7}{12}\right)^2=\frac{529}{144}$ followed by $u\left\{\text{or}\sqrt{x}\right\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\left\{\text{or}\sqrt{x}\right\}=-\frac{5}{2}$ or not.
M1:	Correct method from $p\sqrt{x}\pm q=0$ leading to $x=\ldots$ by squaring In Alt 1, it is for solving their quadratic using the General Principles for Pure Mathematics Marking. There must be a method shown, i.e., the solutions should not come straight from a calculator. If attempting to factorise, it must be to $(ax\pm c)(bx\pm d)$ with $ab=36, cd=400$ In Alt 2, it is for squaring their value(s) for u to get $x=\ldots$
A1:	**cso** $x=\frac{16}{9}$ only. $x=\frac{25}{4}$ must be discarded. Note 0011 is not possible. Allow "incorrect" $x=-\frac{16}{9}$ or $x=-\frac{25}{4}$ to be seen as long as they are discarded. Ignore any reason they give for rejecting solutions. Note that a method to solve their quadratic must be seen – solutions must not come directly from a calculator. Simply stating the quadratic formula (without substitution) is insufficient.

14.

Question	Scheme	Marks	AOs
a	Attempts both $y = 8 - 10\times1 + 6\times1^2 - 1^3$ and $y = 1^2 - 12\times1 + 14$	M1	1.1b
	Achieves $y = 3$ for both equations and gives a minimal conclusion / statement, e.g., $(1, 3)$ lies on both curves so they intersect at $x = 1$	A1	1.1b
		(2)	
(b)	(Curves intersect when) $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ $\Rightarrow x^3 - 5x^2 - 2x + 6 = 0$	M1	1.1b
	For the key step in dividing by $(x-1)$ $x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 + px \pm 6)$	dM1	3.1a
	$x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 - 4x - 6)$	A1	1.1b
	Solves $x^2 - 4x - 6 = 0$ $(x-2)^2 = 10 \Rightarrow x = \ldots$	ddM1	1.1b
	$x = 2 - \sqrt{10}$ only	A1	1.1b
		(5)	
		(7 marks)	

Notes:

(a) **Must be seen in (a)**

M1: As scheme.

For M1 A0, allow a statement that $(1,3)$ lies on both curves without sight of the calculation.

Amongst various alternatives are:

- Setting $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and attempting to rearrange to $x^3 - 5x^2 - 2x + 6 = 0$ before substituting in $x = 1$
- Setting $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and attempting to divide $x^3 - 5x^2 - 2x + 6$ by $(x-1)$ either by long division or inspection

A1: For the complete mathematical argument.
Requires both correct calculations with a minimal conclusion, which may be as a preamble.
e.g., in the alternatives

- as $1^3 - 5\times1^2 - 2\times1 + 6 = 0$, hence curves meet when $x = 1$
- $x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 - 4x - 6)$ so the curves intersect when $x = 1$

(b) **Allow the use of *x* or *k* throughout this part.**

M1: Sets $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and proceeds to a cubic equation set = 0
Must be seen or used in (b)

dM1: For the key step in realising that $(x-1)$ is a factor of the cubic.
It is for dividing by $(x-1)$ to get the quadratic factor.

For division look for their first two terms, i.e., $x^2 \pm 4x$

(This will need checking if they have made an error in rearranging the cubic.)

$$\begin{array}{r} x^2 \pm 4x\ldots\ldots\ldots\ldots \\ x-1\overline{)\,x^3-5x^2-2x+6} \\ \underline{x^3-1x^2} \\ -4x^2 \end{array}$$

By inspection look for the first and last term $x^3-5x^2-2x+6=(x-1)\left(x^2+px\pm 6\right)$

A1: $x^3-5x^2-2x+6=(x-1)\left(x^2-4x-6\right)$ or just x^2-4x-6 or k^2-4k-6 as their quadratic factor following algebraic division.

ddM1: Attempts to solve their $x^2-4x-6=0$, which must be a 3TQ, by completing the square or the quadratic formula, leading to an exact solution. Their quadratic factor must **not** factorise. Their quadratic "factor" may come from algebraic division that has a remainder but we will still allow them to score this mark.

If using the quadratic formula, they need to have, e.g., $\dfrac{4-\sqrt{4^2-4(-6)}}{2}$ or $\dfrac{4-\sqrt{40}}{2}$ as a minimum (i.e., they must not jump straight to $2-\sqrt{10}$ from a calculator).

A1: $k=2-\sqrt{10}$ or exact equivalent but allow the use of x e.g., $x=\dfrac{4-\sqrt{40}}{2}$

If using the quadratic formula, the discriminant must be processed.
Must come from a correct quadratic factor.
They must have discarded $2+\sqrt{10}$ if seen.