Proots As level Edexcle Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
	Considers $\frac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2=3x^2+6xh+3h^2$	M1	1.1b
	so gradient $=\frac{6xh+3h^2}{h}=6x+3h$ or $\frac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h\rightarrow 0$, gradient $\rightarrow 6x$ so in the limit derivative $=6x$ *	A1*	2.5
		(4 marks)	

Notes

B1: gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2=3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Completes the proof, as above (may use $\delta x \rightarrow 0$), considers the limit and states a conclusion with no errors

02.

Question	Scheme	Marks	AOs
a **Way 1**	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes

(a) M1 : Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging.

A1*: Need all three stages making the correct deduction to achieve the printed result.

(b) B1 : Chooses two negative values and substitutes, then states conclusion

03.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0, (x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

....

$x^2-8x+17$

$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

....

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

....

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

....

$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A0

Minimum is (4,1) so $x^2-8x+17>0$ — correct but not explained

....

$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A1

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$ — correct and explained

....

$x^2 - 8x + 17 > 0$
$(x-4)^2 + 1 > 0$ scores M1 A0 (no explanation) A0

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as +ve x^2 etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{dy}{dx} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the candidate just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value.
For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ×)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true.
For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓
or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true and states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x < -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

04.

Question	Scheme	Marks	AOs
	Considers $\frac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0$, $3x^2+3xh+h^2 \to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
		(4 marks)	

Note: On e pen this is set up as B1 M1 M1 A1. We are scoring it B1 M1 A1 A1

B1: Gives the correct fraction for the gradient of the chord either $\frac{(x+h)^3-x^3}{h}$ or $\frac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\frac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x)$, $\frac{dy}{dx}$, y' should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x) \underset{\lim h\to 0}{=} \frac{(x+h)^3-x^3}{h} = 3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x) \underset{\lim h\to 0}{=} 3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0 \Rightarrow f'(x)=3x^2+3x\times 0+0^2=3x^2$

Alternative: B1: Considers $\frac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\frac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

05.

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions. So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8} = \frac{1}{8}n^3 + \frac{3}{8}n^2 + \frac{3}{8}n + \frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"n^3+2 is odd and cannot be divided by 8 **exactly**"
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so n^3+2 is not divisible by 8"
"if n^3 is a multiple of 8 then n^3+2 cannot be divisible by 8
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=8k^3+12k^2+6k+3$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $8k^3+12k^2+6k+3$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes

Correct expressions are required for the M's. There is no need to state **"If n is even,"** $n=2k$ and **"If n is odd,** $n=2k+1$**"** for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

06.

Question	Scheme	Marks	AOs
a	States $(2a-b)^2 .. 0$	M1	2.1
	$4a^2+b^2 .. 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} .. \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} ... 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 .. 0$

A1: Reaches $4a^2+b^2 ... 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab}+\dfrac{b^2}{ab} .. \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\frac{4a}{b} + \frac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

Attempt starting with the left-hand side

M1: $(\text{lhs} =) \frac{4a}{b} + \frac{b}{a} - 4 = \frac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \frac{(2a - b)^2}{ab}$

M1: $= \frac{(2a - b)^2}{ab} \ldots 0$

A1*: Hence $\frac{4a}{b} + \frac{b}{a} - 4 \ldots 0 \Rightarrow \frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

Attempt using given result: For 3 out of 4

$\frac{4a}{b} + \frac{b}{a} .. 4$ M1 $\Rightarrow 4a^2 + b^2 .. 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by ab does not change the sign of the inequality because a and b are positive"

07.

Question	Scheme	Marks	AOs
a	Selects a correct strategy. E.g uses an odd number is $2k\pm1$	B1	3.1a
	Attempts to simplify $(2k\pm1)^3-(2k\pm1)=...$	M1	2.1
	..........and factorise $8k^3\pm12k^2\pm4k=4k(2k^2\pm3k\pm1)=$	dM1	1.1b
	Correct work with statement $4\times..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3-2=6$ which is not a multiple of 4	B1	2.4
		(1)	
		(5 marks)	
Alt (a)	Selects a correct strategy. Factorises $k^3-k=k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that k^3-k is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)

Note: May be in any variable (condone use of n)

B1: Selects a correct strategy. E.g uses an odd number is $2k\pm1$

M1: Attempts $(2k\pm1)^3-(2k\pm1)=...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the k term or the constant of $(2k\pm1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or $4k$ from their cubic

A1: Correct work with statement $4\times..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

08.

Question	Scheme	Marks	AOs
(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1	2.1
	$n(n+1)(n+2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n+1)(n+2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving

- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review

09.

Question	Scheme	Marks	AOs
a	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
		(5 marks)	

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state "not true".
e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.
It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$
Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = \ldots$ condoning slips but must lead to a quadratic.
Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$
May be subtracted the wrong way round for this mark as below.
$(2n)^3 - (2n+2)^3 = \ldots$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$
or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.
Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ **or**, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:
If the even numbers are set as n and $n+2$ there must be sufficient work seen before marks can be awarded.
e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8(3k^2 + 3k + 1)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:
If they just use any two even numbers, e.g., $2a$ and $2b$, **or** $2m$ and $2n+2$ then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8(a^3 - b^3)$ would imply this mark.

A1: $= 8(a^3 - b^3)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers