Projectile Motion A level Edexcel Past Papers Answers

01.

a.

Scheme	Marks	AOs
Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g \leftarrow (3-2)$	M1	3.3
$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
	(2)	

b.

Using the model and horizontal motion: $s = ut$	M1	3.4
$20 = Ut\cos\alpha$	A1	1.1b
Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
sub for U^2	M1(II)	3.1b
$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
$\tan\alpha = \frac{1}{4}$ Þ $\alpha = 14^0$ or better	A1(II)	2.2a
	(9)	

N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
Solve for t	M1(III)	
$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
$\alpha = 14^0$ or better	A1(II)	

b.

ALTERNATIVE		

Using the model and horizontal motion: $s = ut$	M1	3.4
$20 = Ut\cos\alpha$	A1	1.1b
A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$	A1	1.1b
$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b

	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	Þ $\alpha = 14^0$ or better	A1	2.2a
		(9)	
c.	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
d.	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 M1	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 A1	1.1b
		(3)	

d.	**ALTERNATIVE**		
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 M1	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 A1	1.1b
		(3)	
			(15 marks)

Notes:
(a) **M1:** Or any other complete method to obtain an equation in U, g and α **only** **A1*:** Correct GIVEN ANSWER
(b) **M1:** Using horizontal motion **A1:** Correct equation **M1:** Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75 **A1:** Correct equation **M1:** Using $20 = Ut\cos\alpha$ to sub. for t **M1:** Substituting for U^2 using (a) **A1:** Correct quadratic equation (in $\tan\alpha$ **or cot** α**)** **M1:** Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α **A1:** $\alpha = 14^\circ$ or better (No restriction on accuracy since g's cancel) **N.B.** If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark. **(b) ALTERNATIVE** **M1:** Using the model with the usual rules applying to the equation **A1:** Correct equation **M1:** Using the model to obtain the **total** time from A to T **A1:** Correct **total** time t **M1:** Substitute for t in $20 = Ut\cos\alpha$ **M1:** Substitute for U in $20 = Ut\cos\alpha$, using part (a) **A1:** Correct equation in $\tan\alpha$ **only** **M1:** Solve equation for α **A1:** $\alpha = 14^\circ$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for x and y, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark
(c) **B1:** Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their α to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their α to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

02.

	Scheme	Marks	AO
a.	**In this question mark parts (a) and (b) together.**		
	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity <u>at $t = 2$</u>	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	

b.

Using sum of horizontal distances $=50$ at $t=2$	M1	3.3
$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
Vertical distances equal	M1	3.4
$\Rightarrow (20\sin 30°)\times 2 - \frac{g}{2}\times 4 = (u\sin\theta)\times 2 - \frac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
Solving for both θ and u	M1	3.1b
$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
	(6)	

c.

It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
	(1)	
	(13)	

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Marks	Notes
a. B1	Seen or implied, possibly on a diagram
M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.
A1	Correct unsimplified equation in v or v^2
M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component
M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component

b.	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)
	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.
c.	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised

03.

Question	Scheme	Marks	AOs
a.	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	

b.	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45°)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
c.	New value > 28	B1	3.5a
		(1)	
d.	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

Notes:

a.	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation (g does not need to be substituted)
	M1	Must have earned the previous two M marks. Eliminate t and solve for U. **N.B.** They may solve for t first $(100 - \frac{1}{2}gt^2 = -25)$ and then use it to find U.
	A1*	Exact given answer correctly obtained with no wrong working (e.g. $g = 9.81$ used) or approximation seen.
b.	M1	Complete method to give equation in h only (allow if U not substituted), condone

	A1	Correct equation (g does not need to be substituted) (A0 if U is used instead of 28)
c.	A1	cao
	B1	Clear statement
d.	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

04.

Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		
a.	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
b.	Horizontal velocity component at $A = 65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1ft	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 ($m\,s^{-1}$)	A1	1.1b
		(5)	
c.	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
			(10 marks)

Notes

a.	M1	Complete method, correct no. of terms, condone sign errors and sin/cos confusion
	A1	Correct equation in t only with at most one error
	M(A)1	Correct equation in t only
		N.B. For 'up and down' methods etc, the two A marks are for all the equations that they use, lose a mark for each error.
	A1	Cao ($g = 9.8$, 7.1 or 7.11) ($g = 9.81$, 7.1 or 7.12)

b.

B1	Seen, including on a diagram.
M1	Condone sign errors and sin/cos confusion
A1ft	Correct expression; accept negative of this, follow their t
M1	Sub for trig and use Pythagoras
A1	Cao ($g = 9.8$ or 9.81 , 75 or 74.8)

c.

B1	B0 if incorrect extras

05.

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g \times 10$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg \times 10$	A1	1.1b

	ALTERNATIVE 1: If t is time to top: use of $10=\frac{1}{2}gt^2$ oe ($t=\frac{10}{7}$) to obtain an equation in U and α only M1 $U\sin\alpha=14$ or $U\cos\alpha=42$ A1 **ALTERNATIVE 2:** If t is time to top: use of : $10=U\sin\alpha\ t-\frac{1}{2}gt^2$ with $t=\frac{60}{U\cos\alpha}$ substituted to obtain an equation in U and α only : M1 $10=U\sin\alpha\times\frac{60}{U\cos\alpha}-\frac{1}{2}g\left(\frac{60}{U\cos\alpha}\right)^2$ A1		
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha=14 \Rightarrow U\cos\alpha=42$, from part (a) or from using $t=\frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha=20g=196$ and $U^2\sin\alpha\cos\alpha=588$, divide to give $\tan\alpha=\frac{1}{3}$ then $\sin^2\alpha=\frac{1}{10}$, hence result **OR in ALTERNATIVE 2:** sub for U^2 using part (a), to give $\tan\alpha=\frac{1}{3}$ then $\sin^2\alpha=\frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin^2\alpha=\frac{1}{10}$, with no working is DM0A0.		
	$U^2=1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2=1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance</u>' isw	B1	3.5a
		(1)	

5(d)	e.g. wind effects, more accurate value of g, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

Notes:

5a		**N.B.** Could score 2/6 for any one of the 4 given equations if there is no corresponding second equation or there is an attempt but it's incorrect.
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate t, dependent on first and second M1's
	A1*	Given answer correctly obtained, with no wrong working seen. Allow $588 = U^2 \sin\alpha\cos\alpha$ but nothing else
5b	M1	Complete method to give equation in U and α only with correct no. of terms, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate α and rearrange, dependent on first M1
	A1*	Given answer correctly obtained with no wrong working seen (**N.B.** If they use a value for α (18.43.°.) they lose the final A1*)
5c	B1	Clear statement isw
5d	B1	B0 if there is an incorrect extra e.g. mass or weight

06.

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for a.		
a .	Use horizontal motion to give an equation in T and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	
b.	Use vertical motion to give an equation in T and α only	M1	3.3
	$20 = (28\sin\alpha)T - \dfrac{1}{2}gT^2$	A1	1.1b
	Eliminate T to give an unsimplified equation in α only: $20 = (28\sin\alpha)\times\dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1+\tan^2\alpha$ oe to give an unsimplified equation in **tan α only** : $20 = 40\tan\alpha - \dfrac{1}{2}g\times\dfrac{100}{49}(1+\tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	
c.	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \dfrac{3}{\sqrt{10}}$ or $\alpha = 71.565..^o$ to find an equation in H only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3$ $(\alpha = 71.565..^o)$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	

d.	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of *g*, motion takes place in 3D not in 2D, *g* could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	

(11 marks)

Notes

a.	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1*	Correct printed answer correctly obtained. Allow $\frac{10}{7\cos\alpha}=T$ OR $T=\frac{40}{28\cos\alpha}=\frac{10}{7\cos\alpha}$ OR $\frac{40}{28\cos\alpha}=\frac{10}{7\cos\alpha}=T$ OR *t* instead of *T*
b.	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Eliminate *T*, using either the given answer in (a) or their own *T* expression, from their equation to give an unsimplified equation in α only
	M1	Use $\sec^2\alpha=1+\tan^2\alpha$ to produce an equation in $\tan\alpha$ only
	A1*	Given answer correctly obtained. **N.B.** Must be α (or *a*) in the final answer but allow a different angle in the working.
c.	M1	Solve given equation and select larger value of $\tan\alpha$ and use it to try to obtain an equation in *H* only.
	M1	Complete method to give an equation in ***H* only**, using **larger** value of α, correct no. of terms, dim correct, condone sin/cos confusion and sign errors.
	A1	cao. Must be positive, (allow a negative value, changed to a positive answer). **N.B.** This answer comes from use of *g* = 9.8, so must be rounded to 2 or 3 sf.
d.	B1	B0 if any incorrect extras