Logarithm and Exponential As level Edexcel Maths Past Papers Answers

01.

Question	Scheme		Marks	AOs
a	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	Way 1: As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	Way 2: As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	so a = 100 000 **or** b = 1.0116		A1	1.1b
	both a = 100 000 **and** b = 1.0116 (awrt 1.01)		A1	1.1b
			(4)	
(c)	(i) The initial population		B1	3.4
	(ii) The proportional increase of population each year		B1	3.4
			(2)	
(d)	(i) 300000 to nearest hundred thousand		B1	3.4
	(ii) Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	
			(13 marks)	

Notes

(a) M1: Uses a linear equation to relate log P and t
A1: Correct use of gradient and intercept to give a correct line equation

(b) M1: Way 1: Uses logs correctly to give log equation; Way 2 Uses powers correctly to "undo" log equation and expresses as product of two powers
M1: Way 1: Identifies log b or log a or both; Way 2: identifies a or b as powers of 10
A1: Correct value for a or b
A1: Correct values for both

(c) (i) B1: Accept equivalent answers e.g. The population at $t = 0$
(ii) B1: So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year

(d) (i) B1: cao
(ii) M1: as in the scheme A1ft: on their values of a and b with correct log work

(e) As given in the scheme – any two valid reasons

02.

Question	Scheme	Marks	AOs
a	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt (£)2000000	A1	1.1b
		(2)	

(8 marks)

Notes

(a) **This is now being marked M1 A1 M1 A1 and in this order on e pen**

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

.....

ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

.....

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled "p is....." " and "q is ....."

(c)

M1: For substituting $t=30$ into $V=pq^t$ using their values for p and q or substituting $t=30$ into $\log_{10}V=0.05t+4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign. Remember to isw after a correct answer

03.

Question	Scheme	Marks	AOs
a	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

Notes on Questions Continue

A1: 8 years 3 months. Correct answer and solution only
Answers obtained numerically score 0 marks. The M mark must be scored.

(c)
B1: 2 300 but condone £ 2 300

(d)
B1: Any suitable reason. See scheme
Accept "Scrappage" schemes may pay more (or less) than £ 2 300.
Do not accept "does not take into account inflation"
It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

04.

Question	Scheme		Marks	AOs
a	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of ab is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone** $\log_{10}$ **written** log **or** lg **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in a or a correct equation in b

A1: Finds either constant. Allow $a =$ awrt 240 or $b =$ awrt 1.2 following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$
Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

05.

Question	Scheme		Marks	AOs
a	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235" \log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only "3" bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as "122" bpm is not equal to 119 bpm ×

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

06.

Question	Scheme	Marks	AOs
a (i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=...$	M1	1.1b
	$p=-2$	A1	1.1b
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow \log_3\left(\frac{x}{9}\right)^2+\log_3\left(\sqrt{x}\right)^3=-11$ $\Rightarrow \log_3\frac{x^{\frac{7}{2}}}{81}=-11$	M1	1.1b
	$\Rightarrow \frac{x^{\frac{7}{2}}}{81}=3^{-11}$ or equivalent eg $x^{\frac{7}{2}}=3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}}=81\times3^{-11}\Rightarrow x^{\frac{7}{2}}=3^4\times3^{-11}=3^{-7}\Rightarrow x=\left(3^{-7}\right)^{\frac{2}{7}}=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
			(6 marks)

Notes

(a)(i)

B1: Recalls the subtraction law of logs and so obtains $p-2$

(a)(ii)

B1: $\frac{1}{2}p$ oe

(b) ***Be aware this should be solved by non-calculator methods***

M1: Uses their results from part (a) to form a linear equation in p and attempts to solve leading to a value for p. Allow slips in their rearrangement when solving. Allow a misread forming the equation equal to 11 instead of -11

A1: Correct value for p

M1: Uses $\log_3 x=p\Rightarrow x=3^p$ following through on what they consider to be their p. It must be a value rather than p

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Alternative:

M1: Correct use of log rules to achieve an equation of the form $\log_3 ... = \log_3 ...$ or $\log_3 ... = \text{a number}$ (typically -11). Condone arithmetical slips.

A1: Correct equation with logs removed.

M1: Uses inverse operations to find x. Condone slips but look for proceeding from $x^{\frac{a}{b}} = ... \Rightarrow x = ...^{\frac{b}{a}}$ where they have to deal with a fractional power.

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

07.

Question	Scheme	Marks	AOs
a	$h = 2.3 - 1.7e^{0}$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2\times4} = 0.15277...\{\text{m / year}\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
			(6 marks)

Notes:

(a)

M1: Substitutes $t = 0$ into $h = 2.3 - 1.7e^{-0.2t}$ Implied by e.g., $h = 2.3 - 1.7e^{-0}$ or $h = 0.6$

A1: Allow 0.6, 0.6 m, or 60 cm and isw after a correct height. Allow $\frac{3}{5}$

The M mark may be implied by A1.

(b)

M1: Links rate of change to gradient and differentiates $h = 2.3 - 1.7e^{-0.2t}$ to $ke^{-0.2t}$, $k \neq -1.7$

Accept, e.g., $-0.2\times-1.7e^{-0.2t}$ Must be seen in (b).

dM1: Substitutes $t = 4$ into $ke^{-0.2t}$, $k \neq -1.7$ and calculates its value.

A1*: Fully correct. Requires

- sight of $\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$ o.e., e.g., $\left\{\frac{dh}{dt} =\right\} \frac{17}{50}e^{-0.2t}$ or $\left\{\frac{dh}{dt} =\right\} -0.2\times-1.7e^{-0.2t}$
- $\left\{\frac{dh}{dt} =\right\}$ awrt 0.153 {metres per year}
- changing to awrt 15.3 cm {per year}.

Note: Substituting $t = 4$ into $h = 2.3 - 1.7e^{-0.2t}$ gives $h = 1.536...$ scores M0dM0A0 unless differentiation and further correct work is seen separately.

(c)

B1: Allow 2.3, 2.3 m, or 230 cm

$2.2\dot{9}$ and 2.2999… which clearly continues are both acceptable, but 2.29999999 is not.