Logarithm As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
a. way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
b.		(2)	
	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes

(a) M1 : Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging.

A1*: Need all three stages making the correct deduction to achieve the printed result.

(b) B1 : Chooses two negative values and substitutes, then states conclusion

02.

Question	Scheme		Marks	AOs
a.	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
b.	Way 1 $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	Way 2 $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\frac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer.	$x=\frac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	

(4 marks)

Notes

(a) B1: Lists error in line 2 (as above)
B1 : Lists error in line 4 (as above)
(b) M1: Correct work with powers reaching this equation
A1 : Correct answer here – there are many exact equivalents

03.

Question	Scheme		Marks	AOs
a.	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
b.	$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$	$\frac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

(a)

B1: States one of the two errors.

Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law'

Allow responses such as 'it must be $\log x^2$ before subtracting the logs'

Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**

Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0

B1: States both of the two errors. (See above)

Cases like these please send to review.

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x) = 3$

M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right) = b \Rightarrow Ax^n = 2^b$

This is independent of the previous mark so allow following earlier error.

A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0

SC: If the "answer" rather than the "solution" is given score 100.

04.

Question	Scheme	Marks	AOs
a.	For a correct equation in p or q $p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
b.	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
c.	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
		(8 marks)	

Notes

(a) This is now being marked M1 A1 M1 A1 and in this order on e pen

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

....

ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

....

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled "p is............ " and "q is ...."

(c)

M1: For substituting $t=30$ into $V=pq^t$ using their values for p and q or substituting $t=30$ into $\log_{10}V=0.05t+4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

05.

Question	Scheme	Marks	AOs
a.	(£)18 000	B1	3.4
		(1)	
b.	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
c.	2 300	B1	1.1b
		(1)	
d.	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	

(9 marks)

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

Notes on Questions continue

A1: 8 years 3 months. Correct answer and solution only
Answers obtained numerically score 0 marks. The M mark must be scored.

(c)
B1: 2 300 but condone £ 2 300

(d)
B1: Any suitable reason. See scheme
Accept "Scrappage" schemes may pay more (or less) than £ 2 300.
Do not accept "does not take into account inflation"
It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

06.

Question	Scheme		Marks	AOs
a.	Temperature = 83°C		B1	3.4
			(1)	
b.	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
c.	States a suitable reason • As $t \to \infty, \theta \to 18$ from above. • The minimum temperature is 18°C		B1	2.4
			(1)	
d.	$A+B=94$ or $A+Be^{-1}=50$		M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
			(9 marks)	

Notes

(a)

B1: Uses the model to state that the temperature = 83°C Requires units as well

(b)

M1: Uses the information and proceeds to $Pe^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^x and $\ln x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=$ awrt 10.7

(c)

B1: States a suitable reason with minimal conclusion

- As $t \to \infty, \theta \to 18$ from above.
- The minimum temperature is 18°C (so it cannot drop to 15°C)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65e^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $e^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta \neq 15$. All calculations must be correct
- (According to the model) the room temperature is 18°C (so cannot fall below this)

(d)

M1: Attempts to use $(0,94)$ or $(8,50)$ in order to form at least one equation in A and B
Allow this to be scored with an equation containing e^0

A1: Correct equations $A+B=94$ and $A+Be^{-1}=50$ or equivalent. $e^0=1$ must have been processed. Condone $A+B=94$ and $A+0.37B=50$ where $e^{-1}=$ awrt 0.37

dM1: Dependent upon having two equations in A and B formed from the information given. It is a full and correct method leading to a value of A. Allow this to be solved from a calculator.
Note $B=69.6..$ or $\frac{44}{1-e^{-1}} \Rightarrow A=94-"B"$

A1: Deduces that $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4 . Condone $y=\ldots$

07.

Question	Scheme		Marks	AOs
a.	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
b.	The value of *ab* is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
c.	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	

(7 marks)

(a) **Condone** $\log_{10}$ **written** log **or** lg **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in *a* or a correct equation in *b*

A1: Finds either constant. Allow $a =$ awrt 240 or $b =$ awrt 1.2 following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$
Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

08.

Question	Scheme	Marks	AOs
a)	$3x^3-17x^2-6x=0 \Rightarrow x(3x^2-17x-6)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\frac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution …0 to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
		(6 marks)	

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x=0,-\frac{1}{3},6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2=n$ where n is any solution …0 to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2=0 \Rightarrow y=2$

A1ft: Two of $2, 2\pm\sqrt{6}$ but follow through on $(y-2)^2=n \Rightarrow y=2\pm\sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2\pm\sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

09.

Question	Scheme	Marks	AOs
(a)	$35\,(\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60=80-45e^{14c} \Rightarrow 45e^{14c}=20$	M1 A1	1.1b 1.1b
	$\Rightarrow c=\frac{1}{14}\ln\left(\frac{20}{45}\right)=\cdots-0.0579$	dM1	3.1b
	$A=80-45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	

Notes

(a)

B1: Uses the equation of the model to find that $35\,(\text{km}^2)$ of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60=80-45e^{14c} \Rightarrow Ae^{14c}=B$

A1: $45e^{14c}=20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and $\ln x$ are inverse functions and proceeds to a value for c.

A1: Gives a complete equation for the model $A=80-45e^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

10.

Question	Scheme		Marks	AOs
a (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
			(7 marks)	

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only "3" bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as "122" bpm is not equal to 119 bpm ✕

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

11.

Question	Scheme	Marks	AOs
a	$p=10^{0.5}$ (or $\log_{10} p=0.5$) or $q=10^{0.03}$ (or $\log_{10} q=0.03$)	M1	1.1b
	$p=$ awrt 3.162 or $q=$ awrt 1.072	A1	1.1b
	$p=10^{0.5}$ (or $\log_{10} p=0.5$) and $q=10^{0.03}$ (or $\log_{10} q=0.03$)	dM1	3.1a
	$A=3.162\times1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4=\text{"}3.162\text{"}\times\text{"}1.072\text{"}^t$ or $\log_{10}4=0.03t+0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
		(10 marks)	

Notes

(a)

M1: A correct equation in p **or** q. May be implied by a correct value for p or q.
Also score for rearranging the equation to the form $A=10^{0.5}...10^{0.03t}$

A1: For p = awrt 3.162 or q = awrt 1.072. May be embedded within the equation.

dM1: Correct equations in p **and** q. Also score for rearranging the equation to the form $A=10^{0.5}\times10^{0.03t}$

A1: Complete equation with p = awrt 3.162 and q = awrt 1.072. **Must be seen in (a)**
If p and q are just stated but the equation is not written with the values embedded then withhold this mark.
Withhold the final mark if the correct values for p and q result from incorrect working such as $A=10^{0.5}+10^{0.03t}\Rightarrow A=3.162\times1.072^t$.
If p and q are stated the wrong way round, take the stated equation as their final answer and isw.

(b)
(i)
B1: Must reference mass of algae and relating to initially/at the start/beginning
Examples of acceptable answers:
The mass of algae originally (in the pond)
p is the mass of algae when $t=0$
Examples of answers we would not accept
p is how much algae there is at the beginning
The relationship between algae and number of weeks

(ii)

B1: Must reference the rate of change/multiplier and the time frame eg per week/every week/each week.
Examples of acceptable answers:
q is the rate at which the mass of algae increases for every week
The amount of algae increases by 7.2% each week (condone amount for mass in ii)
The proportional increase in mass of the algae each week
Examples of answers we would not accept:
q is how much algae will increase when t increases by 1
The amount that grows per unit of time
The rate at which the mass of algae in the small pond increases after t number of weeks
The rate in which the algae mass increases

(c)

B1: cao (including units)

M1: Setting up a correct equation to find t using the given equation or their part (a)
Substitution of $A = 4$ into their equation for A or the given equation is sufficient for this mark.

A1: awrt 3.4 (weeks). Accept any acceptable method (including trial and improvement)
Condone lack of units. isw if they subsequently convert to weeks and days. Allow awrt 3.5 (weeks) following p = awrt 3.16 and q = awrt 1.07.
An answer of only awrt 3.4 is M1A1, but an answer of 4 (weeks) with no working is M0A0

(d)

B1: Any reason why the rate of change, growth or the mass of algae might change or why the model in not realistic.
Be generous with the awarding of this mark as long as the answer has engaged with the context of the problem or the model
Examples of acceptable answers:
Seasonal changes (which would affect the growth rate)
Overcrowding (as it is a small pond)
Algae may stop growing (the model predicts unlimited growth)
Algae may die / be removed / eaten (so the rate of growth may not continue at the same rate)
Examples of answers we would not accept:
There could be other factors that affect the amount of algae (too vague)
The mass of algae might change

12.

Question	Scheme	Marks	AOs
a)	$(k =)\ 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4e^{-0.5t} \Rightarrow 1.4e^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = ...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{dP}{dt} =\right)\ -0.7e^{-0.5t}$ $\left(\frac{dP}{dt}\right)_{t=2} = -0.7e^{-0.5\times 2}$	M1	3.1b
	$=$ awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

B1: Completes the equation for the model by obtaining $(k =)\ 0.8$ or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with $P = 1$ and their value for k from (a) and proceeding to $Ae^{\pm 0.5t} = B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $Ae^{\pm 0.5t} = B$ leading to a value for t.
They cannot proceed directly to awrt 3.9 without some intermediate working seen.
Eg $t = 2\ln 7$ or $-2\ln\left(\frac{1}{7}\right)$ is acceptable.
Also allow $1.4e^{-0.5t} = 0.2 \Rightarrow -0.5t = -1.9459... \Rightarrow t = ...$
This cannot be scored from an unsolvable equation (eg when their $k \ldots 1$ so that $e^{\pm 0.5t}$,, 0).

A1: Accept awrt 3.9 minutes or $t =$ awrt 3.9 with correct working seen.
eg $1.4e^{-0.5t} = 0.2 \Rightarrow t = 3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $Ae^{-0.5t}$ and substitutes $t = 2$. Do not accept $Ate^{-0.5t}$ as the derivative.
Beware that substituting $t = 2$ and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258
Awrt ± 0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as $P = ...$ or $\frac{dy}{dx} = ...$)

13.

Question	Scheme	Marks	AOs
a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=...$	M1	1.1b
	$p=-2$	A1	1.1b
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow\log_3\left(\frac{x}{9}\right)^2+\log_3\left(\sqrt{x}\right)^3=-11$ $\Rightarrow\log_3\frac{x^{\frac{7}{2}}}{81}=-11$	M1	1.1b
	$\Rightarrow\frac{x^{\frac{7}{2}}}{81}=3^{-11}$ or equivalent eg $x^{\frac{7}{2}}=3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}}=81\times3^{-11}\Rightarrow x^{\frac{7}{2}}=3^4\times3^{-11}=3^{-7}\Rightarrow x=\left(3^{-7}\right)^{\frac{2}{7}}=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(6 marks)	

Notes

(a)(i)

B1: Recalls the subtraction law of logs and so obtains $p-2$

(a)(ii)

B1: $\frac{1}{2}p$ oe

(b) ***Be aware this should be solved by non-calculator methods***

M1: Uses their results from part (a) to form a linear equation in p and attempts to solve leading to a value for p. Allow slips in their rearrangement when solving. Allow a misread forming the equation equal to 11 instead of -11

A1: Correct value for p

M1: Uses $\log_3 x=p\Rightarrow x=3^p$ following through on what they consider to be their p. It must be a value rather than p

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Alternative:

M1: Correct use of log rules to achieve an equation of the form $\log_3 ... = \log_3 ...$ or $\log_3 ... = \text{a number}$ (typically -11). Condone arithmetical slips.

A1: Correct equation with logs removed.

M1: Uses inverse operations to find x. Condone slips but look for proceeding from $x^{\frac{a}{b}} = ... \Rightarrow x = ...^{\frac{b}{a}}$ where they have to deal with a fractional power.

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

14.

Question	Scheme	Marks	AOs
	$2\log_5(3x-2)-\log_5 x=2$		
	Uses one correct law e.g. $2\log_5(3x-2)\to\log_5(3x-2)^2$ **or** $2\to\log_5 25$ **or** $\log_5 ...=2\to ...=5^2$	B1	1.1a
	Uses two correct log laws: either $2\log_5(3x-2)\to\log_5(3x-2)^2$ **and** $2\to\log_5 25$ or $2\log_5(3x-2)-\log_5 x\to\log_5\frac{(3x-2)^2}{x}$ leading to an equation without logs	M1	3.1a
	Correct equation without logs, usually $\frac{(3x-2)^2}{x}=25$	A1	1.1b
	$\frac{(3x-2)^2}{x}=25\Rightarrow 9x^2-37x+4=0\Rightarrow(9x-1)(x-4)=0\Rightarrow x=...$	dM1	1.1b
	$x=4$ only	A1 cso	3.2a
		(5)	

(5 marks)

Notes:

B1: Uses one correct log law. The base does not need to be seen for this mark.
This mark is independent of any other errors they make.

M1: This can be awarded for the overall strategy leading to an equation in x **not involving logs**.
It requires the correct use of two log laws as in the main scheme to reach an equation in x
This mark may **not** be awarded for correct application of two laws following incorrect log work, but numerical slips are condoned.

A1: For a correct unsimplified equation with logs removed and **no incorrect work seen.**
Ignore any incorrect simplification of their equation.

Allow recovery on missing brackets, e.g., $\log_5\frac{3x-2^2}{x}=2\to\frac{9x^2-12x+4}{x}=25$

Correct equations are likely to be $\frac{(3x-2)^2}{x}=25$ or, e.g., $(3x-2)^2=25x$ but you might see

$9x-12+\frac{4}{x}=25$ Sight of a correct equation does **not** imply either the previous M1 or the A1.

Note: $\frac{\log_5(3x-2)^2}{\log_5 x}=2\to\frac{(3x-2)^2}{x}=25$ may be seen and scores B1M0A0.

dM1: For a correct method to solve their equation, **via a 3TQ set = 0**
The 3TQ may be solved by calculator - you may need to check their value(s).
Can be implied by one correct value for their 3TQ set = 0 correct to 1d.p.

A1: cso $x = 4$ only.

If $x = \frac{1}{9}$ is also given it must be rejected. $x = 0$ might also be seen and must be rejected.

Ignore any reasoning for rejecting any values.

Note that calculators can solve the equation at any stage and so full log work must be shown leading to a 3TQ set = 0.

15.

Question	Scheme	Marks	AOs
a	$h = 2.3 - 1.7\text{e}^{0}$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{\text{d}h}{\text{d}t} =\right\} 0.34\text{e}^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34\text{e}^{-0.2\times4} = 0.15277...\{\text{m / year}\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
		(6 marks)	

Notes:

(a)

M1: Substitutes $t = 0$ into $h = 2.3 - 1.7\text{e}^{-0.2t}$ Implied by e.g., $h = 2.3 - 1.7\text{e}^{-0}$ or $h = 0.6$

A1: Allow 0.6, 0.6 m, or 60 cm and isw after a correct height. Allow $\frac{3}{5}$

The M mark may be implied by A1.

(b)

M1: Links rate of change to gradient and differentiates $h = 2.3 - 1.7\text{e}^{-0.2t}$ to $k\,\text{e}^{-0.2t}$, $k \neq -1.7$

Accept, e.g., $-0.2 \times -1.7\text{e}^{-0.2t}$ Must be seen in (b).

dM1: Substitutes $t = 4$ into $k\,\text{e}^{-0.2t}$, $k \neq -1.7$ and calculates its value.

A1*: Fully correct. Requires

- sight of $\left\{\frac{\text{d}h}{\text{d}t} =\right\} 0.34\text{e}^{-0.2t}$ o.e., e.g., $\left\{\frac{\text{d}h}{\text{d}t} =\right\} \frac{17}{50}\text{e}^{-0.2t}$ or $\left\{\frac{\text{d}h}{\text{d}t} =\right\} -0.2 \times -1.7\text{e}^{-0.2t}$
- $\left\{\frac{\text{d}h}{\text{d}t} =\right\}$ awrt 0.153 {metres per year}
- changing to awrt 15.3 cm {per year}.

Note: Substituting $t = 4$ into $h = 2.3 - 1.7\text{e}^{-0.2t}$ gives $h = 1.536...$ scores M0dM0A0 unless differentiation and further correct work is seen separately.

(c)

B1: Allow 2.3, 2.3 m, or 230 cm

$2.2\dot{9}$ and 2.2999… which clearly continues are both acceptable, but 2.29999999 is not.