Integration As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
	$f(x)=2x+3+12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)dx = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes

B1: Correct function with numerical powers
M1: Allow for raising power by one. $x^n \to x^{n+1}$
A1: Correct three terms
M1: Substitutes limits and rationalises denominator
A1*: Completely correct, no errors seen.

02.

Question	Scheme	Marks	AOs
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x=\frac{2}{3}\times\frac{x^4}{4}+\ldots+x$	A1	1.1b
	$=\ldots-6\frac{x^{\frac{3}{2}}}{3/2}+\ldots.$	A1	1.1b
	$=\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n\to x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power**' terms (may be un-simplified at this stage)

A1: Correct '**fractional power**' term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6}-4x\sqrt{x}+1x^1+c$

Accept $\frac{x^4-24x^{\frac{3}{2}}+6x}{6}+c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$

03.

Question	Scheme	Marks	AOs
	For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation • Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis	M1	3.1a
	$y=\frac{32}{x^2}+3x-8\Rightarrow\frac{dy}{dx}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6="-\frac{1}{2}"(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6="-\frac{1}{2}"(x-4)\Rightarrow x=...$ or an attempt using just gradients $"-\frac{1}{2}"=\frac{6}{a-4}\Rightarrow a=...$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{"16"}"\left(-\frac{1}{2}x+8\right)"dx$ The "16" cannot have just been made up.	M1	3.1a
	$\int\frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = = $\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area =10 + 36 =46 *	A1*	2.1
		(10)	
			(10 marks)

(a)
The first 5 marks are for finding the normal to the curve cuts the x - axis
M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme
M1: Differentiates with at least one index reduced by one

A1: $\frac{dy}{dx} = -\frac{64}{x^3} + 3$

dM1: Method of finding
either the equation of the normal at (4, 6) .
or where the equation of the normal at (4, 6) cuts the x - axis
See scheme. It is dependent upon having gained the M mark for differentiation.

A1: Normal cuts the x-axis at $x = 16$

The next 5 marks are for finding the area *R*

M1: For the complete strategy of finding the values of two key areas. See scheme

M1: Integrates $\int \frac{32}{x^2} + 3x - 8\,dx$ raising the power of at least one index

A1: $\int \frac{32}{x^2} + 3x - 8\,dx = -\frac{32}{x} + \frac{3}{2}x^2 - 8x$ which may be unsimplified

dM1: Area $= \left[-\frac{32}{x} + \frac{3}{2}x^2 - 8x\right]_2^4 = (-16) - (-26) = (10)$

It is dependent upon having scored the M mark for integration, for substituting in both 4 and 2 and subtracting either way around. The above line shows the minimum allowed working for a correct answer.

A1*: Shows that the area under curve = 46. No errors or omissions are allowed

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A number of candidates are equating the line and the curve (or subtracting the line from the curve)
The last 5 marks are scored as follows.

M1: For the complete strategy of finding the values of the two key areas. Points that must be seen are

- There must be an attempt to find the area BETWEEN the line and the curve either way around by integrating between 2 and 4
- There must be an attempt to find the area of a triangle using $\frac{1}{2} \times ('16' - 2) \times \left(-\frac{1}{2} \times 2 + 8\right)$ or

  via integration $\int_2^{16} \left("-\frac{1}{2}x + 8"\right) dx$

M1: Integrates $\int \left("-\frac{1}{2}x + 8"\right) - \left(\frac{32}{x^2} + 3x - 8\right) dx$ either way around and raises the power of at least one index by one

A1: $\pm\left(-\frac{32}{x} + \frac{7}{4}x^2 - 16x\right)$ must be correct

dM1: Area $= \int_2^4 \left("-\frac{1}{2}x + 8"\right) - \left(\frac{32}{x^2} + 3x - 8\right) dx =$ ......either way around

A1: Area = $49 - 3 = 46$

NB: Watch for candidates who calculate the area under the curve between 2 and 4 = 10 and subtract this from the large triangle = 56. They will lose both the strategy mark and the answer mark.

NB. Watch for students who use their calculators to do the majority of the work. Please send these items to review

04.

Question	Scheme	Marks	AOs
a	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)dx = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=...$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
		(6 marks)	

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone " unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).
Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ with the indices processed.

A1: Correct (and simplified) with $+c$.
Ignore spurious notation e.g. answer appearing with an $\int$ sign or with dx on the end.
Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$
Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

05.

	Scheme	Marks	AOs
☐	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{dy}{dx} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = ...$	M1	1.1b
	Chooses $x = 4$ ~~$x = \frac{5}{3}$~~	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	
		(7 marks)	

Notes

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.

If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x - \frac{5}{3}\right)$

A1: Chooses $x = 4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**

The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0.

So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.

Expect to see embedded or calculated values.

Don't accept $\int_0^4 2x^3 - 17x^2 + 40x \, dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

Notes On Questions Continue

For correct notation expect to see

- $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y'
- $\int 2x^3 - 17x^2 + 40x \, dx$ used correctly at least once with or without the limits.

06.

Question	Scheme	Marks	AOs
a	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9}$, ~~9~~	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with $+c$ and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

07.

Question	Scheme	Marks	AOs
a	$g(5)=2\times5^3+5^2-41\times5-70=...$	M1	1.1a
	$g(5)=0\Rightarrow(x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3+x^2-41x-70=(x-5)(2x^2...x\pm14)$	M1	1.1b
	$=(x-5)(2x^2+11x+14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x))=(x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3+x^2-41x-70\,dx=\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5}g(x)dx$ $-\frac{1525}{3}-\frac{190}{3}$	M1	2.2a
	Area = $571\frac{2}{3}$	A1	2.1
		(4)	
		(10 marks)	

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0
Look for evidence of embedded values or two correct terms of
$g(5)=250+25-205-70=...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,
$g(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$
$g(5)=0\Rightarrow(x-5)$ is a factor ✓
Do not allow if candidate states
$f(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**
$g(x)=0\Rightarrow(x-5)$ is a factor **(It is not g(x) and there is no conclusion)**
This may be seen in a preamble before finding $g(5)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2+11x+14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2+11x+14)$

A1: $(\mathrm{g}(x)=)\ (x-5)(2x+7)(x+2)$ or $(\mathrm{g}(x)=)\ (x-5)(x+3.5)(2x+4)$
It is for the product of factors and not just a statement of the three factors
Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \to x^{n+1}$ for any of the terms in x for $\mathrm{g}(x)$ so
$2x^3 \to ...x^4,\ x^2 \to ...x^3,\ -41x \to ...x^2,\ -70 \to ...x$

A1: $\int 2x^3 + x^2 - 41x - 70\,\mathrm{d}x = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to area = $571\frac{2}{3}$ oe

So allow $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,\mathrm{d}x = \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5} = -\frac{1715}{3} \Rightarrow$ area $= \frac{1715}{3}$ for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x$ then withhold the final mark if they just write a positive value to this integral since $\int_{-2}^{5} \mathrm{g}(x)\mathrm{d}x = -\frac{1715}{3}$

Note $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,\mathrm{d}x \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

08.

Question	Scheme	Marks	AOs
	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

09.

Question	Scheme	Marks	AOs
9	$\int_k^9 \frac{6}{\sqrt{x}}\,dx = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36 - 12\sqrt{k} = 20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36 - 12\sqrt{k} = 20 \Rightarrow k =$	dM1	3.1a
	$\Rightarrow k = \frac{16}{9}$ oe	A1	1.1b
		(4)	
		(4 marks)	

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36 - 12\sqrt{k} = 20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}} < 0$

A1: $k = \frac{16}{9}$

10.

Question	Scheme	Marks	AOs
a	$f(x)=-3x^2+12x+8=-3(x\pm2)^2+...$	M1	1.1b
	$=-3(x-2)^2+...$	A1	1.1b
	$=-3(x-2)^2+20$	A1	1.1b
		(3)	
(b)	Coordinates of $M=(2,20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2+12x+8\,dx=-x^3+6x^2+8x$	M1 A1	1.1b 1.1b
	Method to find $R=$ their $2\times20-\int_0^2(-3x^2+12x+8)\,dx$	M1	3.1a
	$R=40-[-2^3+24+16]$	dM1	1.1b
	$=8$	A1	1.1b
		(5)	
		(10 marks)	
Alt(c)	$\int 3x^2-12x+12\,dx=x^3-6x^2+12x$	M1 A1	1.1b 1.1b
	Method to find $R=\int_0^2 3x^2-12x+12\,dx$	M1	3.1a
	$R=2^3-6\times2^2+12\times2$	dM1	1.1b
	$=8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x\pm2)^2+...$
Alternatively attempt to compare $-3x^2+12x+8$ to $ax^2+2abx+ab^2+c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2+...$ or via comparison finds $a=-3, b=-2$

A1: $-3(x-2)^2+20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $(-b, c)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{"2"} f(x)\, dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

11.

Question	Scheme	Marks	AOs
	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)dx = \frac{8x^4}{4} ... + 5x$	A1	1.1b
	$= ... - 2\times\frac{3}{2}x^{\frac{1}{2}} + ...$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: For raising any correct power of x by 1 including $5 \to 5x$ (not for $+c$) Also allow eg $x^3 \to x^{3+1}$

A1: For 2 correct non-fractional power terms (allow unsimplified coefficients) and may appear on separate lines. The indices must be processed. The $+c$ does not count as a correct term here. Condone the 1 appearing as a power or denominator such as $\frac{5x^1}{1}$ for this mark.

A1: For the correct fractional power term (allow unsimplified) Allow eg $+-2\times 1.5\sqrt{x^1}$.

Also allow fractions within fractions for this mark such as $\frac{\frac{3}{2}}{\frac{1}{2}}x^{\frac{1}{2}}$

A1: All correct and simplified and on one line including $+c$. Allow $-3\sqrt{x}$ or $-\sqrt{9x}$ for $-3x^{\frac{1}{2}}$.

Do not accept $+-3x^{\frac{1}{2}}$ for this mark.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int 2x^4 - 3x^{\frac{1}{2}} + 5x + c \, dx$ or $2x^4 - 3x^{\frac{1}{2}} + 5x + c = 0$ with no correct expression seen earlier are both A0.

12.

Question	Scheme	Marks	AOs
a	$y=\frac{1}{3}x^2-2\sqrt{x}+3 \Rightarrow \frac{dy}{dx}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4 \Rightarrow y=\frac{13}{3}$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=4}=\frac{2}{3}\times4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right)\quad \therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1	2.1
	$13x-6y-26=0$*	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)dx=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1	1.1b 1.1b
	$y=0 \Rightarrow x=2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-"2")\times"\frac{13}{3}"=\frac{76}{9}-\frac{13}{3}$	M1	3.1a
	$=\frac{37}{9}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a) **Calculators: If no algebraic differentiation seen then maximum in a) is M0A0B1M1A0***

M1: $x^n \to x^{n-1}$ seen at least once $...x^2 \to ...x^1$, $...x^{\frac{1}{2}} \to ...x^{-\frac{1}{2}}$, $3 \to 0$.

Also accept on sight of eg $...x^{\frac{1}{2}} \to ...x^{\frac{1}{2}-1}$

A1: $\frac{2}{3}x-x^{-\frac{1}{2}}$ or any unsimplified equivalent (indices must be processed) accept the use of $0.\dot{6}x$ but not rounded or ambiguous values eg $0.6x$ or eg $0.66....x$

B1: Correct y coordinate of P. May be seen embedded in an attempt of the equation of l

M1: Fully correct strategy for an equation for l. Look for $y-"\frac{13}{3}"="\frac{13}{6}"(x-4)$ where their $\frac{13}{6}$ is from differentiating the equation of the curve and substituting in $x=4$ into their $\frac{dy}{dx}$ and the y coordinate is from substituting $x=4$ into the given equation.
If they use $y=mx+c$ they must proceed as far as $c=...$ to score this mark.
Do not allow this mark if they use a perpendicular gradient.

A1*: Obtains the printed answer with no errors.

(b) **Calculators: If no algebraic integration seen then maximum in b) is M0A0B1M1A0**

M1: $x^n \to x^{n+1}$ seen at least once. Eg $...x^2 \to ...x^3$, $...x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$, $3 \to 3x^1$. Allow eg $...x^2 \to ...x^{2+1}$ The $+c$ is not a valid term for this mark.

A1: $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x$ or any unsimplified equivalent (indices must be processed) accept the use of exact decimals for $\frac{1}{9}$ $(0.\dot{1})$ and $-\frac{4}{3}$ $(-1.\dot{3})$ but not rounded or ambiguous values.

B1: Deduces the correct value for x for the intersection of l with the x-axis. May be seen indicated on Figure 2.

M1: Fully correct strategy for the area. This needs to include

- a correct attempt at the area of the triangle using their values (**could use integration**)
- a correct attempt at the area under the curve using 0 and 4 in their integrated expression
- the two values subtracted.

Be aware of those who mix up using the y-coordinate of P and the gradient at P which is M0. The values embedded in an expression is sufficient to score this mark.

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.111…. isw after a correct answer

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Be aware of other strategies to find the area *R*

eg Finding the area under the curve between 0 and 2 and then the difference between the curve and the straight line between 2 and 4:

$$\int_0^2 \frac{x^2}{3} - 2\sqrt{x} + 3 \; dx + \int_2^4 \frac{x^2}{3} - 2\sqrt{x} - \frac{13}{6}x + \frac{22}{3} \; dx$$

M1 $x^n \to x^{n+1}$ seen at least once on either integral (or on the equation of the line $y = \frac{1}{3}x + 3$)

A1 for correct integration of **either** integral $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x$ or $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} - \frac{13}{12}x^2 + \frac{22}{3}x$ (may be unsimplified/uncollected terms but the indices must be processed with/without the $+C$)

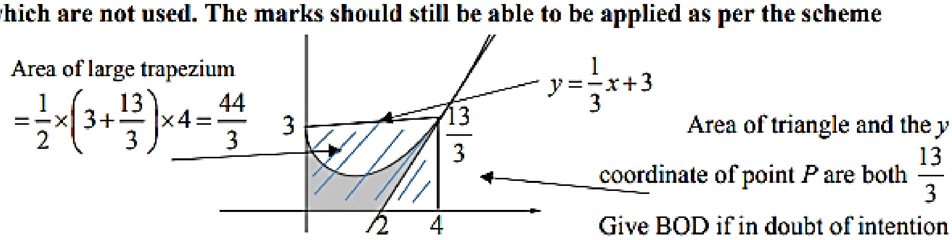
B1 Correct value for x can be seen from the top of the first integral (or bottom value of the second integral)

M1 Correct strategy for the area eg.

$$\left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x\right]_0^2 + \left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} - \frac{13}{12}x^2 + \frac{22}{3}x\right]_2^4 = \frac{62}{9} - \frac{4}{3}(2)^{\frac{3}{2}} + \frac{76}{9} - \frac{101}{9} + \frac{4}{3}(2)^{\frac{3}{2}}$$

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.1 or 4.111….

You could also see use of the area of a trapezium and/or the use of the line $y = \frac{1}{3}x + 3$ to find other areas which could be combined or used as part of the strategy to find *R*. Ignore areas which are not used. The marks should still be able to be applied as per the scheme



Area of trapezium – (Area between $y = \frac{1}{3}x + 3$ and curve C + area of triangle)

$$= \frac{44}{3} - \frac{56}{9} - \frac{13}{3} = \frac{37}{9}$$

13.

Question	Scheme	Marks	AOs
	States or uses the upper limit is $\sqrt{5}$	B1	1.1b
	$\int 4x^2+3\,dx = \frac{4}{3}x^3+3x$	M1 A1	1.1b 1.1b
	Full method of finding the area of R e.g. $23\sqrt{5}-\left[\frac{4}{3}x^3+3x\right]_0^{\sqrt{5}}=...$ e.g. $\left[20x-\frac{4}{3}x^3\right]_0^{\sqrt{5}}=...$	M1	2.1
	$\Rightarrow$ Area $R=\frac{40}{3}\sqrt{5}$	A1	1.1b
		(5)	
		(5 marks)	

Notes:

B1: States or uses the upper limit $\sqrt{5}$ Score when seen as the solution $x=\sqrt{5}$

M1: Attempts to integrate $4x^2+3$ **or** $\pm\left(23-\left(4x^2+3\right)\right)$ which may be simplified.

Look for one term from $4x^2+3$ with $x^n \to x^{n+1}$ It is not sufficient just to integrate 23.

A1: Correct integration. Ignore any $+c$ or spurious integral signs. Indices must be processed.

Look for $\int 4x^2+3\{dx\}=\frac{4}{3}x^3+3x$ **or** $\pm\int 20-4x^2\{dx\}=\pm\left(20x-\frac{4}{3}x^3\right)$ if (line – curve) or (curve – line) used.

M1: Full and complete method to find the area of R including the substitution of their upper limit. The upper limit must come from an attempt to solve $4x^2+3=23$

The lower limit might not be seen but if seen it should be 0.

See scheme for two possible ways. Condone a sign slip if (line – curve) or (curve – line) used.

A1: $\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

If using (curve – line) then allow recovery but they must make the $-\frac{40}{3}\sqrt{5}$ positive.

Alternative using $\int x\,dy$

B1: States or uses limits 3 and 23. It must be for a clear attempt to integrate with respect to y

M1: Attempts to rearrange to $x=$ and integrate $\sqrt{\frac{y-3}{4}}$ condoning slips on the rearrangement.

Look for $...(y\pm3)^{\frac{1}{2}} \to ...(y\pm3)^{\frac{3}{2}}$

A1: Correct integration $\int \frac{(y-3)^{\frac{1}{2}}}{2}\{dy\}=\frac{1}{3}(y-3)^{\frac{3}{2}}$ Ignore any $+c$ or spurious integral signs.

M1: Full and complete method to find the area of R including the substitution of their limits. In this case it would be for substituting 23 and 3 and subtracting either way round into their changed expression in terms of y

A1: $\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

14.

Question	Scheme	Marks	AOs
	Sets $f'(4)=0 \Rightarrow 16+2a+b=0$	M1	2.1
	Integrates $f'(x)=4x+a\sqrt{x}+b \Rightarrow \{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx\ \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c=-5$	B1	2.2a
	Full and complete method using the given information $f'(4)=0$ and $f(4)=3$ in order to find values for a and b Note: $a=-15$ and $b=14$	ddM1	3.1a
	$\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$	A1	1.1b
		(6)	
		(6 marks)	

Notes:

M1: For the key step in setting $f'(4)=0 \Rightarrow 16+2a+b=0$ to set up an equation in a and b. Condone slips.

M1: For attempting to integrate $f'(x)$. Award for $x^n \to x^{n+1}$ or $b \to bx$
This may come after finding values for a or b or both.

A1ft: $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx\ \{+c\}$ or, e.g., $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+(-16-2a)x\ \{+c\}$
Allow ft on their b in terms of a if they substituted in from their $f'(4)=0 \Rightarrow 16+2a+b=0$
Do not ft if they have a value(s) for a or b
This may be left unsimplified but the indices must be processed.
isw once the mark is awarded. Condone the omission of the $+c$
This accuracy mark requires only the previous M mark to be scored.

B1: Deduces that the constant term in $f(x)$ is -5.
Note that deducing $b=-5$ is B0. It must be the constant in a changed function.

ddM1: For a complete strategy to find values for both a and b.
Do not be concerned about the logistics of how they solve the simultaneous equations – this may be done on a calculator.
Note: $a=-15$ and $b=14$
This is dependent on **both** previous method marks and so must include use of both
- $f'(4)=0$ (their $16+2a+b=0$ o.e.)
- $f(4)=3$ (their $32+\frac{16}{3}a+4b-5=3$ o.e.)

A1: $\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$ or exact simplified equivalent, e.g., use of $x\sqrt{x}$ in place of $x^{\frac{3}{2}}$
Apply isw once a correct expression is seen.