Inequalities As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes 3 = 0 (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k = 0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

02.

Question	Scheme	Marks	AOs
2(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0, (x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

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$x^2-8x+17$
$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$
Hence $(x-4)^2+1>0$

scores M1 A1 A1

$x^2-8x+17>0$
$(x-4)^2+1>0$
This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

$x^2-8x+17>0$
$(x-4)^2+1>0$
which is true because a squared number is positive

scores M1 A1 A0
incorrect and incomplete

$x^2-8x+17=(x-4)^2+1$
Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0
correct but not explained

$x^2-8x+17=(x-4)^2+1$
Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1
correct and explained

$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$

scores M1 A0 (no explanation) A0

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as +ve x^2 etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{dy}{dx} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the candidate just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value. For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ×)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true. For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true and states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x < -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

03.

Question	Scheme	Marks	AOs
a	$(g(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
			(9 marks)

Notes

(a)

M1: Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3 - 12\times(-2)^2 - 15\times(-2) + 50$ embedded

Or $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2) = 0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$g(-2) = 0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \ldots\ldots\ldots \pm 25)$

If algebraic / long division is used expect to see $x+2 \overline{)\ 4x^3 - 12x^2 - 15x + 50}$ with quotient $4x^2 \pm 20x$

A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d),\ ac = \pm 4, bd = \pm 25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $g(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x = 2.5$ Follow through on their $g(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $g(x) = (x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

....

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

....

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either *a* or *b*	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

04.

Question	Scheme	Marks	AOs
a	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2+b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\frac{4a^2}{ab}+\frac{b^2}{ab} \geqslant \frac{4ab}{ab}$	M1	2.2a
	Hence $\frac{4a}{b}+\frac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \frac{4a}{b}+\frac{b}{a}=-20-\frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
		(5 marks)	

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2+b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab}+\frac{b^2}{ab} \geqslant \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a,b>0$ such that $\frac{4a}{b}+\frac{b}{a}<4$

A1: $4a^2+b^2<4ab \Rightarrow 4a^2+b^2-4ab<0$

M1: $(2a-b)^2<0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b}+\frac{b}{a}\ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

Attempt starting with the left-hand side

M1: $(\text{lhs}=)\frac{4a}{b}+\frac{b}{a}-4=\frac{4a^2+b^2-4ab}{ab}$

A1: $=\frac{(2a-b)^2}{ab}$

M1: $=\frac{(2a-b)^2}{ab}\ldots 0$

A1*: Hence $\frac{4a}{b}+\frac{b}{a}-4\ldots 0 \Rightarrow \frac{4a}{b}+\frac{b}{a}\ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a>0$ and $b>0$

Attempt using given result: For 3 out of 4

$\frac{4a}{b}+\frac{b}{a}..4$ M1 $\Rightarrow 4a^2+b^2 ..4ab \Rightarrow 4a^2+b^2-4ab\ldots 0$

A1 $\Rightarrow (2a-b)^2\ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by ab does not change the sign of the inequality because a and b are positive"

05.

Question	Scheme	Marks	AOs
	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

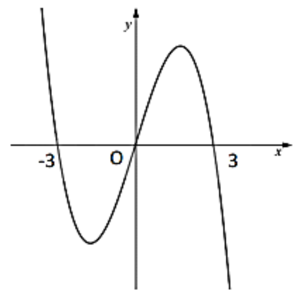
M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4,\ x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

06.

Question	Scheme	Marks	AOs
a	$9x - x^3 = x(9 - x^2)$	M1	1.1b
	$9x - x^3 = x(3 - x)(3 + x)$ oe	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	B1	1.1b
	Passes though origin, (3, 0) and (−3, 0)	B1	1.1b
		(2)	
(c)	$y = 9x - x^3 \Rightarrow \dfrac{dy}{dx} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = ...$	M1	3.1a
	$y = (\pm)6\sqrt{3}$	A1	1.1b
	$\{k \in \square : -6\sqrt{3} < k < 6\sqrt{3}\}$ oe	A1ft	2.5
		(3)	
			(7 marks)

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x(\pm x \pm 3)(\pm x \pm 3)$.
Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x(3 - x)(3 + x)$ on its own scores M1A1.
Allow eg $-x(x - 3)(x + 3)$, $x(x - 3)(-x - 3)$ or other equivalent expressions
Condone an = 0 appearing on the end and condone eg x written as $(x + 0)$.

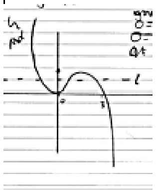
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

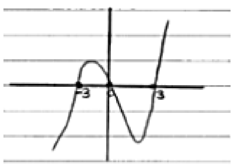
B1: Passes **through** each of the origin, (3, 0) and (−3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and −3 on the axes. Condone x and y to be the wrong way round eg $(0, -3)$ for $(-3, 0)$ as long as it is on the correct axis but do not allow (−3, 0) to be labelled as (3, 0).

Examples

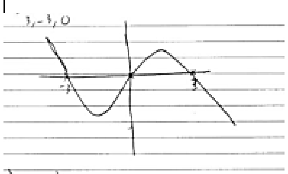
B1B0

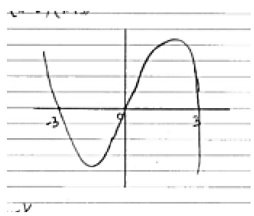


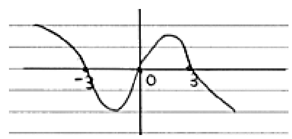
B0B1



B1B1







(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{dy}{dx} = 0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

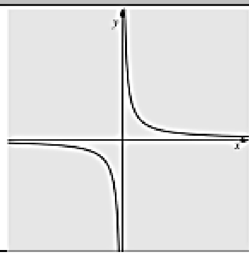
A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\{"-6\sqrt{3}" < k < "6\sqrt{3}"\}$ or $\{"-6\sqrt{3}" < k\} \cap \{k < "6\sqrt{3}"\}$ but not $\{"-6\sqrt{3}" < k\} \cup \{k < "6\sqrt{3}"\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of k

07.

Question	Scheme		Marks	AOs
a		Shape in quadrant 1 **or** 3	M1	1.1b
		Shape and Position	A1	1.1b
			(2)	
(b)	Deduces that $x<0$		B1	2.2a
	Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$		M1	1.1b
	$x<0$ or $x \geqslant 8$		A1 cso	2.2a
			(3)	
			(5 marks)	

Notes:

(a)

M1: For the correct shape in quadrant 1 **or** 3. Do not be concerned about position but it must not cross either axis. Ignore incorrect asymptotes for this mark.

A1: Correct shape and position. There should be no curve in either quadrant 2 or quadrant 4. The curve must not clearly bend back on itself but condone slips of the pen.

(b)

B1: Deduces that $x<0$ but condone $x \leqslant 0$ for this mark.

M1: Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$ where the … means any equality or inequality.

A1: cso $x<0$ or $x \geqslant 8$ (Both required)

Set notation may be seen $\{x:x<0\}\cup\{x:x\geqslant 8\}$ o.e. $x\in(-\infty,0)\cup[8,\infty)$

Accept $x<0, x \geqslant 8$ but not $x<0$ and $x \geqslant 8$

Must not be combined incorrectly, e.g., $8 \leqslant x<0$ or $\{x:x<0\}\cap\{x:x\geqslant 8\}$

08.

Question	Scheme	Marks	AOs
8	Complete method to find the RHS of an equation for l e.g., Attempts gradient $= \frac{80-60}{10}\{=2\}$ and uses intercept = 60	M1	1.1b
	$\{y=\}2x+60$	A1	1.1b
	Deduces the RHS of the equation for C is $\{y=\}ax(x-6)$ and attempts to use $(10,80)$ to find the value of a	M1	3.1a
	Equation of C is $\{y=\}2x(x-6)$	A1	1.1b
	$2x(x-6) \leqslant y \leqslant 2x+60$	B1ft	2.5
		(5)	
		(5 marks)	

Notes:

M1: Complete attempt to use the given information to find an equation or inequality for l, which may be $l =$ or have no LHS. $y - 80 = 2(x - 10)$ is acceptable for this mark.

A1: $\{y=\}2x+60$ This is not scored by $y - 80 = 2(x - 10)$

M1: Deduces the RHS of the equation of C is $\{y=\}ax(x-6)$, $a \neq 1$, and attempts to use $(10,80)$ to find the value of a which may be implied. Again, there may be no LHS.
Other possible and more lengthy alternatives include:
1) Setting the RHS to be $\{y=\}a(x-3)^2+b$ and using $(0,0)$ and $(10,80)$ to find a and b
2) Setting the RHS to be $\{y=\}px^2+qx$ and using $(6,0)$ and $(10,80)$ to find p and q

A1: $\{y=\}2x(x-6)$ or alternative such as $\{y=\}2(x-3)^2-18$ or $\{y=\}2x^2-12x$
This may be implied by an inequality $y \ldots 2x(x-6)$ and may be seen as, e.g., $C=2x(x-6)$

B1ft: $"2x(x-6)" \leqslant y \leqslant "2x+60"$ o.e. must follow from their l and C and apply isw
Follow through only on a quadratic for C and a straight line for l
Do not allow a mixture of inequalities, i.e., $<$ with $\leqslant$
Allow $2x^2-12x < y < 2x+60$ or as separate inequalities $y > 2x(x-6)$, $y < 2x+60$
Do not allow $2x(x-6) < R < 2x+60$ **or** $2x(x-6) < f(x) < 2x+60$ **or** $2x(x-6) < 2x+60$
Ignore any reference to $-3 < x < 10$
Note: $y=2x+60$ and $y=2x(x-6)$ incorrectly expanded to $y=2x^2-12$ followed by $2x^2-12 \leqslant y \leqslant 2x+60$ would score 11110