Differentiation As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
☐	Attempt to differentiate	M1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	M1	1.1b
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 8$	A1ft	1.1b
			(4 marks)

Notes

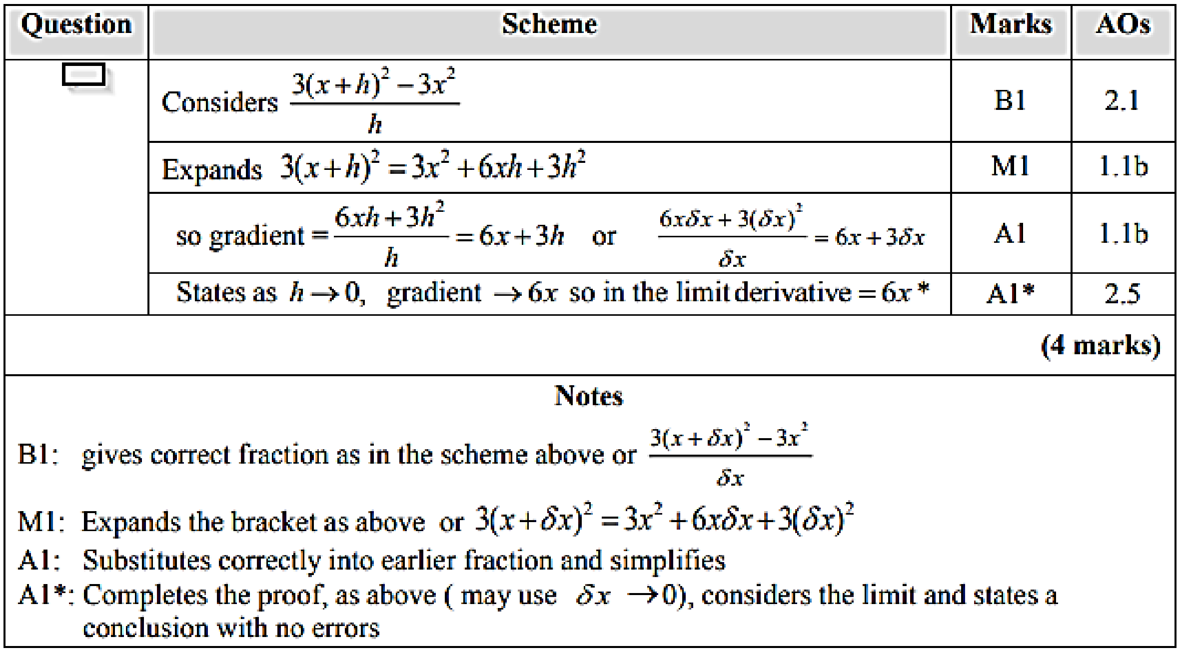
M1 : Differentiation implied by one correct term

A1 : Correct differentiation

M1 : Attempts to substitute $x = 5$ into their derived function

A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their f ′(5) so follow through slips in differentiation

02.



Question	Scheme	Marks	AOs
	Considers $\frac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2=3x^2+6xh+3h^2$	M1	1.1b
	so gradient $=\frac{6xh+3h^2}{h}=6x+3h$ or $\frac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h\to 0$, gradient $\to 6x$ so in the limit derivative $=6x$ *	A1*	2.5
			(4 marks)

Notes

B1: gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2=3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

03.

Question	Scheme	Marks	AOs
a	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8m.	A1	1.1b
		(4)	
		(10 marks)	

Notes

(a) B1 : Correct area equation

M1 : Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P

M1 : Use correct equation for perimeter with their y substituted

A1*: Completely correct solution to obtain and state printed answer

(b) M1 : States $x>0$ and $y>0$ and uses their expression from (a) to form inequality

A1*: Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly.

(c) M1: Attempt to differentiate P (deals with negative power of x correctly)

A1 : Correct differentiation

M1 : Sets derived function equal to zero and obtains $x=$

A1: The value of x may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$).

Need to see awrt 59.8m with units included for the perimeter.

04.

Question	Scheme	Marks	AOs
$\boxed{a}$)(i)	$C=\frac{1500}{v}+\frac{2v}{11}+60 \Rightarrow \frac{\mathrm{d}C}{\mathrm{d}v}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0 \Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\,(\mathrm{km\,h^{-1}})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=+\frac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	

(9 marks)

Notes

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{\mathrm{d}C}{\mathrm{d}v}$ in the form $\frac{A}{v^2}+B$

A1: $\left(\frac{\mathrm{d}C}{\mathrm{d}v}\right)=-\frac{1500}{v^2}+\frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k, k>0$

Allow here equations of the type $\frac{1}{v^n}=k, k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 ($\mathrm{km\,h^{-1}}$) . Don't be concerned by incorrect / lack of units.

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\frac{\mathrm{d}C}{\mathrm{d}v}$ appearing as $\frac{\mathrm{d}y}{\mathrm{d}x}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding $C =$ from their solution to $\frac{dC}{dv} = 0$.

Do not accept attempts using negative values of v.
Award if you see $v = ..$, $C = ...$ where the v used is their solution to (a)(i). You do not need to check this calculation.
A1ft: Minimum cost = awrt (£) 93. Condone the omission of units
Follow through on sensible values of v. $60 < v < 110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{d^2C}{dv^2}$ (following through on their $\frac{dC}{dv}$ which must be of equivalent difficulty) and attempts to find its value / sign at their v

Allow a substitution of their answer to (a) (i) in their $\frac{d^2C}{dv^2}$

Allow an explanation into the sign of $\frac{d^2C}{dv^2}$ from its terms (as $v > 0$)

A1ft: $\frac{d^2C}{dv^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{d^2C}{dv^2} = +\frac{3000}{v^3} > 0$ as $v > 0$

Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their v ($v > 0$) and their $\frac{d^2C}{dv^2}$

* Condone $\frac{d^2C}{dv^2}$ appearing as $\frac{d^2y}{dx^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)
B1: Gives a limitation of the given model, for example

- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable. Do not accept/ignore irrelevant statements such as "air resistance" etc

05.

Question	Scheme	Marks	AOs
a	$(\mathrm{g}(-2)) = 4\times-8-12\times4-15\times-2+50$	M1	1.1b
	$\mathrm{g}(-2)=0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$	M1 A1	1.1b 1.1b
	$=(x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x=2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x=-1,\ x=1.25$	B1ft	2.2a
		(3)	

(9 marks)

Notes

(a)

M1: Attempts $\mathrm{g}(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3-12\times(-2)^2-15\times(-2)+50$ embedded

Or $-32-48+30+50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $\mathrm{g}(-2)=0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$\mathrm{g}(-2)=0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $\mathrm{g}(-2)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to divide $\mathrm{g}(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3-12x^2-15x+50=(x+2)(4x^2............\pm25)$

If algebraic / long division is used expect to see

$$\begin{array}{r} 4x^2\pm20x \\ x+2\overline{\big)\ 4x^3-12x^2-15x+50} \end{array}$$

A1: Correct quadratic factor is $(4x^2-20x+25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2-20x+25)$ usual rule $(ax+b)(cx+d), ac=\pm4, bd=\pm25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $\mathrm{g}(x)=(x+2)(x-2.5)^2=(x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x=2.5$ Follow through on their $\mathrm{g}(x)=(x+2)(ax+b)^2$ only where $ab<0$ (that is a positive root). Condone $x<-2$ See SC below for $\mathrm{g}(x)=(x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $\mathrm{g}(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $\mathrm{g}(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $\mathrm{g}(x) = (x+2)(ax+b)^2$

……

SC: If a candidate reaches $\mathrm{g}(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

……

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either *a* or *b*	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

06.

Question	Scheme	Marks	AOs
a	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = 6x - \dfrac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \dfrac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\dfrac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. $+2$ for this mark

The indices now must have been processed

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x - \dfrac{24}{x^2}$ or exact simplified equivalent. Eg accept $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x^1 - 24x^{-2}$

You do not need to see the $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\dfrac{\mathrm{d}y}{\mathrm{d}x}...0$ and proceeds to $x...$ via an allowable intermediate equation

$\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p...q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

07.

Question	Scheme	Marks	AOs
	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
		(5 marks)	

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$

It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

08.

Question	Scheme	Marks	AOs
a	Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=2$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x\Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ=\frac{3(2+h)^2-2-10}{(2+h)-2}$ oe	B1	1.1b
	$=\frac{3(2+h)^2-12}{(2+h)-2}=\frac{12h+3h^2}{h}$	M1	1.1b
	$=12+3h$	A1	2.1
		(3)	
(c)	Explains that as $h\to 0,\ 12+3h\to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	

(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2-2\to ...x$ and substitutes $x=2$ into their answer

A1: cso $\frac{\mathrm{d}y}{\mathrm{d}x}=6x\Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12+3h$

(c)

B1: Explains that as $h\to 0,\ 12+3h\to 12$ and states that the gradient of the chord tends to the gradient of the curve

09.

Question	Scheme	Marks	AOs
a	$V=\pi r^2h=355\Rightarrow h=\frac{355}{\pi r^2}$ $\left(\text{or } rh=\frac{355}{\pi r} \text{ or } \pi rh=\frac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\frac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\frac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\frac{dC}{dr}=0.26\pi r-\frac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\frac{dC}{dr}=0\Rightarrow r^3=\frac{28.4}{0.26\pi}\Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\frac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\frac{d^2C}{dr^2}=\right)0.26\pi+\frac{56.8}{r^3}=0.26\pi+\frac{56.8}{"3.26"^3}$	M1	1.1b
	$\left(\frac{d^2C}{dr^2}=\right)(2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi("3.26")^2+\frac{28.4}{"3.26"}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	

(12 marks)

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\frac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see

- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355=\pi r^2 h$ and $C=0.04\pi r^2+0.09\pi r^2+0.04\times 2\pi rh=0.13\pi r^2+0.08\pi\times\left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1}\rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}r}=0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2\rightarrow ...r$ or $...r^{-1}\rightarrow ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ at their (positive) r or considers the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$.

This mark can be scored as long as their second derivative is of the form $A+\frac{B}{r^3}$ where A and B are non zero

A1: Requires

- A correct $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$
- Either
  - deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}>0$ for $r>0$ (without evaluating). There must be some minimal explanation as to why it is positive.
  - substitute their positive r into $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ without evaluating and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}>0$ for $r>0$
  - evaluate $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ (which must be awrt 2.5) and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}>0$ for $r>0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C=)\ 13$ ignore units

10.

Question	Scheme	Marks	AOs
a	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}2x^2-7x-4$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts to solve $\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}2x^2-7x-4\ldots0$ e.g., $(2x+1)(x-4)=0$ leading to $x=\ldots$ and $x=\ldots$	M1	1.1b
	Correct critical values $x=-\frac{1}{2},4$	A1	1.1b
	Chooses inside region for their critical values	dM1	1.1b
	Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\leqslant x\leqslant 4$	A1	1.1b
		(4)	

(6 marks)

Notes:

(a)

M1: Decreases the power of x by one for at least one of their terms. Look for $x^n \rightarrow \ldots x^{n-1}$

Allow for $5 \rightarrow 0$

A1: $\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}2x^2-7x-4$

(b)

M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}\ldots0$ where … may be an equality or an inequality and proceeds to find two values for x from a 3TQ using the usual rules. This may be implied by their critical values.

A1: Correct critical values $x\ldots-\frac{1}{2},4$

These may come directly from a calculator and might only be seen on a sketch.

dM1: Chooses the inside region for their critical values.

A1: Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\leqslant x\leqslant 4$ but not, e.g., $-\frac{1}{2}<x\leqslant 4$

Condone, e.g., $x>-\frac{1}{2},\ \ x<4$ **or** $x>-\frac{1}{2}$ and $x<4$ **or** $\left\{x:x>-\frac{1}{2}\right\}\cap\{x:x<4\}$

or $x\in\left(-\frac{1}{2},4\right)$ **or** $x\in\left[-\frac{1}{2},4\right]$

Note: You may see $x<-\frac{1}{2},x<4$ in their initial work before $-\frac{1}{2}<x<4$. Condone this so long as it is clear that the $-\frac{1}{2}<x<4$ is their final answer.