Circle As level Edexcle Maths Past Papers Answers

01.

Question	Scheme		Marks	AOs
a.	Way 1: Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	Way 2: Finds distance between $(-2,6)$ and $(10, 11)$	M1	3.1a
	Checks whether (10,1) satisfies their circle equation	Finds distance between $(-2,6)$ and (10, 1)	M1	1.1b
	Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that (10,1) lies on C *	Concludes that as distance is the same (10, 1) lies on the circle C *	A1*	2.1
			(3)	
b.	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or -23		A1	1.1b
	Way 1: Deduces gradient of second tangent	Way 2: Deduces midpoint of PQ from symmetry ((0,6))	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ=35+23=58*		A1*	1.1b
			(7)	

(10 marks)

Notes

(a) Way 1 and Way 2:

M1 : Starts to use information in question to find equation of circle or radius of circle

M1 : Completes method for checking that (10, 1) lies on circle

A1*: Completely correct explanation with no errors concluding with statement that circle passes through (10, 1)

(b) M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ or $\frac{12}{5}$) This is referred to as m' in the next note.

M1: Attempts $y - 11 = their\left(-\frac{12}{5}\right)(x-10)$ or $y - 1 = their\left(\frac{12}{5}\right)(x-10)$ and puts $x = 0$, or uses vectors to find intercept e.g. $\frac{y-11}{10} = -m'$

A1: One correct intercept 35 or -23 (continued on next page)

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Qu b) continued

Way 1:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept

e.g. $\frac{11-y}{10} = m'$

Way 2:

M1: Finds midpoint of PQ from symmetry. (This is at (0,6))

M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. $35 - 6 = 29$ then $6 - 29 = -23$ so second intercept is at $(-23, 0)$

Ways 1 and 2:

A1*: Obtain 58 correctly from a valid method.

02.

Question	Scheme	Marks	AOs
a.	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
b.	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their *a*, *b* and *c* leading to values for *k* "$(10k-6)^2-36(1+k^2)...0$" $\to k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
		(9 marks)	

Notes

(a)
M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$
This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$
(i) A1: Centre $(3,-5)$
(ii) A1: Radius 5. Do not accept $\sqrt{25}$
Answers only scores all three marks

(b)
B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant0$ or even with greater than symbols
M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just *x* and *k*. It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.
A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of *a*, *b* and *c* are used in $b^2-4ac...0$.
FYI The equation in *y* and *k* is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe
M1: Attempts to find two critical values for *k* using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.

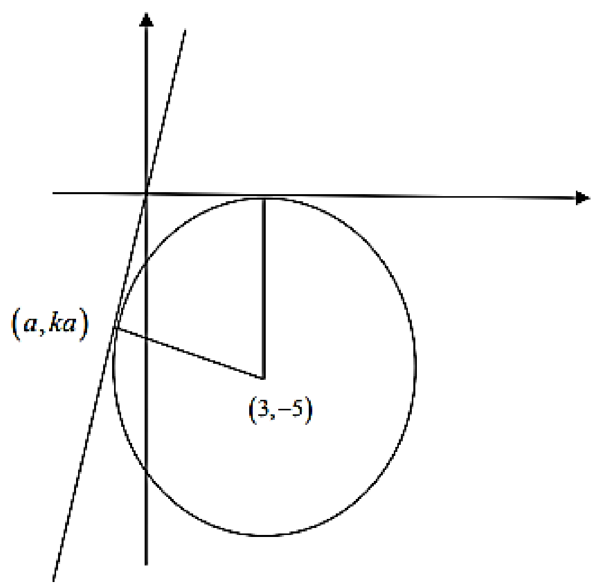
$b^2 - 4ac = (k-6)^2 - 4\times(1+k^2)\times 9 > 0 \Rightarrow -35k^2 - 12k > 0 \Rightarrow k(35k+12) < 0$

A1: Deduces $k < 0, k > \frac{15}{8}$. This must be in terms of k.

Allow exact equivalents such as $k < 0 \cup k > 1.875$

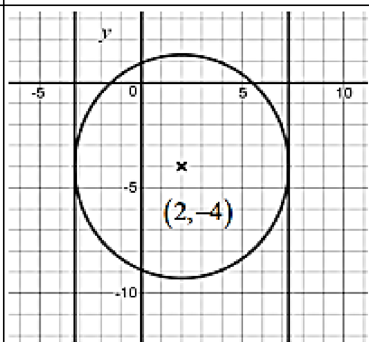
but not allow $0 > k > \frac{15}{8}$ or the above with AND, & or $\cap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$



Alt (b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Distance from (a,ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2) = 9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2) = 3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k = \frac{15}{8}$	A1	1.1b
	Deduces $k < 0, k > \frac{15}{8}$	A1	2.2a
		(6)	

03.

Question	Scheme	Marks	AOs
(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract 'r' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	

(5 marks)

Notes

(a)

M1: Attempts to complete the square. Look for $(x\pm2)^2+(y\pm4)^2 ...$

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm2,\pm4)$ Condone $a=\pm2, b=\pm4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds x/k by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root.

E.g. $b^2-4ac=0 \Rightarrow 64-4(k^2-4k-8)=0 \Rightarrow k= ..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm5.29 \rightarrow k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

04.

Question	Scheme	Marks	AOs
□. (i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" = $\frac{7-1}{-5+9}=\left(\frac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\frac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
		(9 marks)	

Notes

(i)

M1: Attempts $(x\pm9)^2....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\frac{7-\text{"1"}}{-5-\text{"}-9\text{"}}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\frac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in\mathbb{Z}$

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\frac{dy}{dx}+18-2\frac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\frac{dy}{dx}+18-2\frac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\frac{dy}{dx}$

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P-k}$

M1: Applying the strategy that it lies entirely within quadrant if "their radius" < 4 and proceeding to obtain an inequality in k only (See scheme). Condone ... „ 4 for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k$ „ 52

05.

Question	Scheme	Marks	AOs
(a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)

(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point P. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets C using a vector approach

M1: For a full method leading to k. Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both b and c are dependent upon k.** The terms in x^2 and x must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in k.
Award if you see use of correct formula but it would be implied by $\pm$ correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding k

A1: $k=\frac{10}{3}$ only

06.

Question	Scheme	Marks	AOs
(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0\Rightarrow y=-\frac{1}{2}x+...\Rightarrow-\frac{1}{2}\to 2$	B1	2.2a
	$m_N=2\Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0\Rightarrow x=...,\ y=...$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left("5"-"\frac{6}{5}"\right)^2+\left("4"+"\frac{18}{5}"\right)^2}-"3"$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

M1: Attempts to complete the square for both x and y terms $(x\pm5)^2$ .... $(y\pm4)^2$ which may be implied by a centre of (±5, ±4)

A1: Centre (5, 4)

A1: Radius 3

(b)

B1: Deduces the gradient of the perpendicular to l is 2. May be seen in the equation for the perpendicular line to l

M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of l
Look for $y-"4"="2"(x-"5")$ where $(5,4)$ is their centre being solved simultaneously with the equation of l
Do not be concerned with the mechanics of their rearrangement when solving simultaneously.
Many are finding the y-intercept of l $(0,-3)$ which is M0

A1: $\left(\frac{6}{5}, -\frac{18}{5}\right)$ or equivalent eg $(1.2, -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between l and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5}, \ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left("\frac{25-3\sqrt{5}}{5}" - "\frac{6}{5}"\right)^2 + \left("\frac{20-6\sqrt{5}}{5}" + "\frac{18}{5}"\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|"5"\times"1"+"4"\times"2"+"6"|}{\sqrt{"1"^2+"2"^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}} \left(= \frac{19}{\sqrt{5}}\right)$

dM1: Distance $= "\frac{19\sqrt{5}}{5}" - 3$

A1: $\frac{19\sqrt{5}-15}{5}$

07.

Question	Scheme	Marks	AOs
a	$x^2+y^2-6x+10y+k=0$		
	$(x-3)^2+(y+5)^2\pm...=...$	M1	1.1b
	Centre $(3,-5)$	A1	1.1b
		(2)	
(b)	Deduces that $k=9$ is a critical point	B1ft	2.2a
	Recognises that radius > 0 $"9"+"25"-k>0$	M1	3.1a
	$9<k<34$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: For sight of $(x\pm3)^2\pm(y\pm5)^2\pm...=...$ or one coordinate for centre from $(\pm3,\pm5)$

A1: Centre $(3,-5)$

(b)

B1ft: Deduces that $k...9$ is a critical point. Allow this to come from their $("5")^2$ Condone $\frac{36}{4}$

Note that this might come from setting $y=0$ and using the discriminant on $x^2-6x+k=0$

M1: $(x\pm3)^2+(y\pm5)^2=("3")^2+("5")^2-k$ **and** recognises that the radius^2 must be positive so $("3")^2+("5")^2-k>0$ but condone $("3")^2+("5")^2-k\geqslant0$

$k<34$ **or** $k\leqslant34$ would imply this method mark.

Note: they may have incorrectly calculated $("3")^2+("5")^2$ in (a) so allow their value for this in place of $("3")^2+("5")^2$ as long as the intention is clear.

A1: $9<k<34$ but condone $9<k\leqslant34$. Allow inequalities to be separate, i.e., $k>9, k<34$

Set notation may be seen $\{k:k>9\}\cap\{k:k<34\}$ **or** $k\in(9,34)$

Condone $\{k:k>9\}\cap\{k:k\leqslant34\}$ **or** $k\in(9,34]$ **or** $k>9$ and $k\leqslant34$

Must not be combined incorrectly, e.g., $\{k:k>9\}\cup\{k:k<34\}$