Binomial Expansion As level Edexcel Maths Past Papers Answers

01.

Question	Scheme	Marks	AOs
a	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right) + \binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128 + ...$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... -224x + ...$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... + ... + 168x^2 \ (+ ...)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
		(5 marks)	

Notes

(a) M1: Need correct binomial coefficient with correct power of 2 and correct power of x.
Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent
B1: Correct answer, simplified as given in the scheme.
A1: Correct answer, simplified as given in the scheme.
A1: Correct answer, simplified as given in the scheme.

(b) B1: Needs a full explanation i.e. to state $x = 0.01$ **and** that this would be substituted **and** that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

02.

Question	Scheme	Marks	AOs
a	$\left(2-\frac{x}{16}\right)^9 = 2^9 + \binom{9}{1}2^8.\left(-\frac{x}{16}\right)+\binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512 + ...$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... -144x + ...$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... + ... + 18x^2 (+ ...)$	A1	1.1b
		(4)	
(b)	Sets $'512'a = 128 \Rightarrow a = ...$	M1	1.1b
	$(a=)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b + '-144'a = 36 \Rightarrow b = ...$	M1	2.2a
	$(b=)\frac{9}{64}$ oe	A1	1.1b
		(2)	
			(8 marks)
a alt	$\left(2-\frac{x}{16}\right)^9 = 2^9\left(1-\frac{x}{32}\right)^9 = 2^9\left(1+\binom{9}{1}\left(-\frac{x}{32}\right)+\binom{9}{2}\left(-\frac{x}{32}\right)^2+...\right)$	M1	1.1b
	$= 512 + ...$	B1	1.1b
	$= ... -144x + ...$	A1	1.1b
	$= ... + ... + 18x^2 (+ ...)$	A1	1.1b

Notes Mark (a)(b) and (c) as one complete question

(a)

M1: Attempts the binomial expansion. May be awarded on either term two and/or term three Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $\left(\pm\frac{x}{16}\right)$ Condone $\binom{9}{2}2^7.\left(-\frac{x^2}{16}\right)$ for term three.

Allow any form of the binomial coefficient. Eg $\binom{9}{2} = {}^9C_2 = \frac{9!}{7!2!} = 36$

In the alternative it is for attempting to take out a factor of 2 (may allow 2^n outside bracket) and having a correct binomial coefficient combined with a correct power of $\left(\pm\frac{x}{32}\right)$

B1: For 512
A1: For $-144x$

A1: For $+\ 18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks

(b)
M1: For setting their $512a = 128$ and proceeding to find a value for *a*. Alternatively they could substitute $x = 0$ into both sides of the identity and proceed to find a value for *a*.

A1 ft: $a = \frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)
M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of "a" to find a value for "b"

A1: $b = \frac{9}{64}$ oe

03.

Question	Scheme	Marks	AOs
a	2^6 or 64 as the constant term	B1	1.1b
	$\left(2+\frac{3x}{4}\right)^6 = ...+{}^6C_1 2^5\left(\frac{3x}{4}\right)^1 + {}^6C_2 2^4\left(\frac{3x}{4}\right)^2 + ...$ $= ...+6\times 2^5\left(\frac{3x}{4}\right)^1 + \frac{6\times 5}{2}\times 2^4\left(\frac{3x}{4}\right)^2 + ...$	M1 A1	1.1b 1.1b
	$= 64+144x+135x^2+...$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4} = -0.075 \Rightarrow x = -0.1$ So find the value of $64+144x+135x^2$ with $x=-0.1$	B1ft	2.4
		(1)	
		(5 marks)	

Notes

(a)

B1: Sight of either 2^6 or 64 as the constant term

M1: An attempt at the binomial expansion. This may be awarded for a correct attempt at either the second **OR** third term. Score for the correct binomial coefficient with the correct power of 2 and the correct power of $\frac{3x}{4}$ condoning slips. Correct bracketing is not essential for this M mark.

Condone ${}^6C_2 2^4 \frac{3x^2}{4}$ for this mark

A1: Correct (unsimplified) second **AND** third terms.

The binomial coefficients must be processed to numbers /numerical expression e.g $\frac{6!}{4!2!}$ or $\frac{6\times 5}{2}$

They cannot be left in the form ${}^6C_1$ and/or $\binom{6}{2}$

A1: $64+144x+135x^2+...$ Ignore any terms after this. Allow to be written $64, 144x, 135x^2$

(b)

B1ft: $x=-0.1$ or $-\frac{1}{10}$ **with** a comment about substituting this into their $64+144x+135x^2$

If they have written (a) as $64, 144x, 135x^2$ candidate would need to say substitute $x=-0.1$ into the sum of the first three terms.

As they do not have to perform the calculation allow

Set $2+\frac{3x}{4}=1.925$, solve for x and then substitute this value into the expression from (a)

If a value of x is found then it must be correct

Alternative to part (a)

$$\left(2+\frac{3x}{4}\right)^6 = 2^6\left(1+\frac{3x}{8}\right)^6 = 2^6\left(1+{}^6C_1\left(\frac{3x}{8}\right)^1 + {}^6C_2\left(\frac{3x}{8}\right)^2 + ...\right)$$

B1: Sight of either 2^6 or 64

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term. Score for the correct binomial coefficient with the correct power of $\frac{3x}{8}$ Correct bracketing is not essential for this mark.
A1: A correct attempt at the binomial expansion on the second and third terms.
A1: $64 + 144x + 135x^2 + ...$ Ignore any terms after this.

04.

Question	Scheme	Marks	AOs
a	$(1+kx)^{10}=1+\binom{10}{1}(kx)^1+\binom{10}{2}(kx)^2+\binom{10}{3}(kx)^3...$	M1 A1	1.1b 1.1b
	$=1+10kx+45k^2x^2+120k^3x^3...$	A1	1.1b
		(3)	
(b)	Sets $120k^3=3\times10k$	B1	1.2
	$4k^2=1\Rightarrow k=...$	M1	1.1b
	$k=\pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

(a)

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term or fourth term. The coefficients may be of the form $^{10}C_1$, $\binom{10}{2}$ etc or eg $\frac{10\times9\times8}{3!}$

A1: A correct unsimplified binomial expansion. The coefficients must be numerical so cannot be of the form $^{10}C_1$, $\binom{10}{2}$. Coefficients of the form $\frac{10\times9\times8}{3!}$ are acceptable for this mark.

The bracketing must be correct on $(kx)^2$ but allow recovery

A1: $1+10kx+45k^2x^2+120k^3x^3...$ or $1+10(kx)+45(kx)^2+120(kx)^3...$
Allow if written as a list.

(b)

B1: Sets their $120k^3=3\times$ their $10k$ (Seen or implied)
For candidates who haven't cubed allow $120k=3\times$ their $10k$
If they write $120k^3x^3=3\times$ their $10kx$ only allow recovery of this mark if x disappears afterwards.

M1: Solves a cubic of the form $Ak^3=Bk$ by factorising out/cancelling the k and proceeding correctly to at least one value for k. Usually $k=\sqrt{\frac{B}{A}}$

A1: $k=\pm\frac{1}{2}$ o.e ignoring any reference to 0

05.

Question	Scheme	Marks	AOs
a	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \dfrac{243}{32}$	M1	1.1b
	$\Rightarrow a = \dfrac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	$= 256 + 5670 = 5926$	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket ${}^8C_5 2^3 ax^5$ and left without the binomial coefficient expanded

A1: $448a^5x^5$ Allow unsimplified but ${}^8C_5$ must be "numerical"

M1: Sets their $448a^5 = 3402$ and proceeds to $\Rightarrow a^k = ...$ where $k \in \mathbb{N}$ $k \neq 1$

A1: Correct work leading to $a = \dfrac{3}{2}$

(b)

M1: Finds either term required. So allow for 2^8 or ${}^8C_4 2^4 a^4$ (even allowing with a)

dM1: Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$

A1: cso 5926

06.

Question	Scheme	Marks	AOs
6(a)	3^8 or 6561 as the constant term	B1	1.1b
	$\left(3-\frac{2x}{9}\right)^8 = ...+{}^8C_1(3)^7\left(-\frac{2x}{9}\right)+{}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2+{}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3+...$ $=...+8\times(3)^7\left(-\frac{2x}{9}\right)+28\times(3)^6\left(-\frac{2x}{9}\right)^2+56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b
	$=6561-3888x+1008x^2-\frac{448}{3}x^3+...$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{2}\times\text{"}1008\text{"}-\frac{1}{2}\times\text{"}-\frac{448}{3}\text{"}$	M1	3.1a
	$=\frac{1736}{3}$ (or $578\frac{2}{3}$)	A1	1.1b
		(2)	
		(6 marks)	

Notes
(a)

B1: Sight of 3^8 or 6561 as the constant term.

M1: An attempt at the binomial expansion. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of 3 and the correct power of $(\pm)\frac{2x}{9}$. Condone invisible brackets eg ${}^8C_2(3)^6-\frac{2x}{9}^2$ for this mark.

A1: For a correct simplified or unsimplified **second** or **fourth term** (with binomial coefficients evaluated).

$+8\times(3)^7\left(-\frac{2x}{9}\right)$ or $+56(3)^5\left(-\frac{2x}{9}\right)^3$

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$. Do not isw if they multiply all the terms by eg 3

Alt(a)

B1: Sight of $3^8(1+.....)$ or 6561 as the constant term

M1: An attempt at the binomial expansion $\left(1-\frac{2}{27}x\right)^8$. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of $(\pm)\frac{2x}{27}$. Condone invisible brackets for this mark.

Score for any of:
$8\times-\frac{2}{27}x$, $\frac{8\times7}{2}\times\left(-\frac{2}{27}x\right)^2$, $\frac{8\times7\times6}{6}\times\left(-\frac{2}{27}x\right)^3$ which may be implied by any of
$-\frac{16}{27}x$, $+\frac{112}{729}x^2$, $-\frac{448}{19683}x^3$

A1: For a correct simplified or unsimplified **second** or **fourth** term including being multiplied by 3^8

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$

(b)

M1: Adopts a correct strategy for the required coefficient. This requires an attempt to calculate $\pm$ ½ their coefficient of x^2 from part (a) $\pm$ ½ their coefficient of x^3 from part (a).

There must be an attempt to bring these terms together to a single value. ie they cannot just circle the relevant terms in the expansion for this mark. The strategy may be implied by their answer.
Condone any appearance of x^2 or x^3 appearing in their intermediate working.

A1: $\frac{1736}{3}$ or $578\frac{2}{3}$ Do not accept $578.\dot{6}$ or $\frac{1736}{3}x^2$

07.

Question	Scheme	Marks	AOs
	Attempts the term in x^3 or the term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ or ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$	M1	3.1a
	Correct term in x^3 or correct term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ $-\frac{135}{2}x^3$ or $-\frac{9}{16}x^5$	A1	1.1b
	Attempts one of the required terms in x^5 of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$ Either $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ or $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	M1	1.1b
	Attempts the sum of $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	dM1	2.1
	Coefficient of x^5 $=-\frac{45}{16}-540=-\frac{8685}{16}$	A1	1.1b
		(5)	

(5 marks)

Notes:

M1: For the key step in attempting to find one of the required terms in the expansion of $\left(3-\frac{1}{2}x\right)^6$ to enable the problem to be solved.

Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ or ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ but condone missing brackets and slips in signs.

May be part of a complete expansion but only one of the required terms needs to be of the correct form.

A1: For $-\frac{135}{2}\{x^3\}$ or $-\frac{9}{16}\{x^5\}$ which may be unsimplified but the ${}^6C_3$ or ${}^6C_5$ must be processed. May be implied by $-540\{x^5\}$ or $-\frac{45}{16}\{x^5\}$

M1: Attempts one of the required terms in x^5 of the expansion of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$

Look for $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ or $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ which would also imply the previous M.

The x^5 may be missing as just the coefficient is required.

May be implied by $-540\{x^5\}$ or $-\frac{45}{16}\{x^5\}$

Condone missing brackets and signs.

You might see candidates make a slip in, e.g., their binomial coefficients, but have an (essentially) correct method to solve the problem.

Note that this M mark is not dependent on the first, so you may be able to award it even if they have made a slip in finding their x^3 or x^5 term in the expansion.

dM1: Attempts the sum of $5\times {}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times {}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$

Dependent on the previous M but may be scored at the same time.

The x^5 may be missing as just the coefficients are required.

Condone missing brackets and signs.

A1: $-\frac{8685}{16}$ or exact equivalent, -542.8125 and apply isw

Condone $-\frac{8685}{16}x^5$ for A1

Note that rounded decimals, e.g., -542.81 will not score the last mark.

Note that full marks can be scored for concise solutions such as:

$$5\times {}^6C_5\times 3\times\left(-\frac{1}{2}\right)^5 \quad + \quad 8\times {}^6C_3\times 3^3\times\left(-\frac{1}{2}\right)^3 \quad = -\frac{8685}{16}$$

Alternative

Attempts via the taking out of the common factor can be scored in the same way.

$$\left(3-\frac{1}{2}x\right)^6 = 3^6\left\{1+6\times\left(-\frac{1}{6}x\right)^1+\frac{6\times5}{2}\left(-\frac{1}{6}x\right)^2+\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3+\frac{6\times5\times4\times3}{4!}\left(-\frac{1}{6}x\right)^4+\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5+\left(-\frac{1}{6}x\right)^6\right\}$$

For M1 A1 look for $3^6\times\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3$ or $3^6\times\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5$

Score the remaining marks as per the main scheme.