Level 2 Certificate
FURTHER MATHEMATICS
8365/2

Paper 2 Calculator

Mark scheme

June 2022

Version: 1.0 Final

226G8365/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
[a, b)	Accept values between a and b including a but excluding b.
(a, b]	Accept values between a and b excluding a but including b.
(a, b)	Accept values between a and b excluding both a and b.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	$6w(2 + 3w)$	B2	B1 partial factorisation eg $3w(4 + 6w)$ or $2w(6 + 9w)$ or $w(12 + 18w)$ or $6(2w + 3w^2)$ or $3(4w + 6w^2)$ or $2(6w + 9w^2)$

Additional Guidance

B1 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
$6w(2 + 3w)$ seen with further simplification	B1
$1(12w + 18w^2)$	B0
$(2 + 3w)$ is equivalent to $(3w + 2)$ etc	
Condone $(6w + 0)$ for $6w$ etc	
Condone use of multiplication signs in B2 or B1 responses eg $6w \times (2 + 3w)$ or $w \times 6 \times (2 + 3w)$	B2
Condone missing closing bracket in B2 or B1 responses eg $3w(4 + 6w$	B1
Condone $w3$ for $3w$ etc for B1 eg $w3(4 + 6w)$	B1
$w6(2 + 3w)$	B1
Ignore any attempt to 'solve' after B2 or B1 seen	
Responses involving fractions or surds are not acceptable	

Q	Answer	Mark	Comments
2	–8	B2	B1 correct equation or calculation eg $\frac{a+6}{2} = -1$ or $a + \frac{6-a}{2} = -1$ or –1 – 7 or 6 – 7 × 2 allow a to be any letter
	Additional Guidance		
	Answer –8 (no need to check working)		B2
	Accept (…, –8) or …, –8 (no need to check working)		B2
	Allow working in vectors eg $\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix}$		B1
	oe equations may involve equation of the line eg1 grad = $\frac{6--1}{8-2}$ $y = \frac{7}{6}x - \frac{10}{3}$ $y = \frac{7}{6} \times -4 - \frac{10}{3}$ eg2 $-1 - a = \frac{7}{6}(2 - -4)$		 B1 B1

Q	Answer	Mark	Comments
3(a)	**Alternative method 1** Starts by multiplying 1st matrix by 3		
	$\begin{pmatrix} 12 & 6 \\ 3 & 0 \end{pmatrix}$	B1	brackets may be missing but values must be in correct position in a 2 by 2 array
	At least two values correct from evaluation of their $\begin{pmatrix} 12 & 6 \\ 3 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}$	M1	brackets may be missing but values must be in correct position in a 2 by 2 array multiplication of matrices must be in the order shown
	$\begin{pmatrix} 18 & 30 \\ 6 & 0 \end{pmatrix}$	A1ft	must have brackets ft B0M1
	Alternative method 2 Starts by multiplying the matrices		
	At least two values correct in $\begin{pmatrix} 6 & 10 \\ 2 & 0 \end{pmatrix}$	M1	brackets may be missing but values must be in correct position in a 2 by 2 array
	$\begin{pmatrix} 6 & 10 \\ 2 & 0 \end{pmatrix}$	A1	brackets may be missing but values must be in correct position in a 2 by 2 array
	$\begin{pmatrix} 18 & 30 \\ 6 & 0 \end{pmatrix}$	B1ft	must have brackets ft 3 × their $\begin{pmatrix} 6 & 10 \\ 2 & 0 \end{pmatrix}$ their $\begin{pmatrix} 6 & 10 \\ 2 & 0 \end{pmatrix}$ must be a 2 by 2 array

Additional Guidance is on the next page

3(a) cont	**Additional Guidance**	
	Alt 1 $\begin{pmatrix}12 & 6\\3 & 0\end{pmatrix}\begin{pmatrix}2 & 0\\-1 & 5\end{pmatrix} = \begin{pmatrix}18 & 42\\6 & 8\end{pmatrix}$	B1M1A0ft
	Alt 1 $\begin{pmatrix}12 & 6\\3 & 0\end{pmatrix}\begin{pmatrix}2 & 0\\-1 & 5\end{pmatrix} = \begin{pmatrix}24 & 35\\4 & 0\end{pmatrix}$	B1M0A0ft
	Alt 1 $\begin{pmatrix}12 & 6\\1 & 0\end{pmatrix}\begin{pmatrix}2 & 0\\-1 & 5\end{pmatrix} = \begin{pmatrix}18 & 30\\2 & 0\end{pmatrix}$	B0M1A1ft
	Alt 1 $\begin{pmatrix}7 & 5\\4 & 3\end{pmatrix}\begin{pmatrix}2 & 0\\-1 & 5\end{pmatrix} = \begin{pmatrix}14 & 25\\5 & 20\end{pmatrix}$	B0M1A0ft
	Alt 2 $\begin{pmatrix}6 & 10\\1 & 5\end{pmatrix}$ with answer $\begin{pmatrix}18 & 30\\3 & 15\end{pmatrix}$	M1A0B1ft
	Alt 2 $\begin{pmatrix}8 & 0\\-1 & 0\end{pmatrix}$ with answer $\begin{pmatrix}24 & 0\\-3 & 0\end{pmatrix}$	M0A0B1ft
	Alt 2 $\begin{pmatrix}8 & 0\\-1 & 0\end{pmatrix}$ with answer $\begin{pmatrix}24 & 0\\-1 & 0\end{pmatrix}$	M0A0B0ft
	For the final mark allow if there is intention to enclose the correct elements in brackets	
	Responses that start by multiplying 2nd matrix by 3 should be marked using the principles of Alt 1	
	Multiplying both matrices by 3 can score a maximum of B1 $\begin{pmatrix}12 & 6\\3 & 0\end{pmatrix}$ or $\begin{pmatrix}6 & 0\\-3 & 15\end{pmatrix}$	B1M0A0ft

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th colspan="2">Comments</th></tr>
<tr><td rowspan="11">3(b)</td><td>$14 + a^3 = 78$
or
$2b - 5a = 12$
or
$14 + a^3$ and $2b - 5a$</td><td>M1</td><td colspan="2">oe eg $a^3 = 64$
or $2b + -5a = 12$
allow eg $7 \times 2 + a^2 \times a$ for $14 + a^3$
allow eg $2 \times b - 5 \times a$ for $2b - 5a$</td></tr>
<tr><td>$a = 4$</td><td>A1</td><td colspan="2"></td></tr>
<tr><td>$\dfrac{12 + 5 \times \text{their}\,a}{2}$ correctly evaluated</td><td>A1ft</td><td colspan="2">accept an exact value or a value rounded to 1 dp or better</td></tr>
<tr><td colspan="4">Additional Guidance</td></tr>
<tr><td colspan="3">$\begin{pmatrix} 14 + a^3 \\ 2b - 5a \end{pmatrix}$ or $(14 + a^3, 2b - 5a)$ with or without brackets</td><td>M1</td></tr>
<tr><td colspan="3">$a = 4$ (M1 is implied)</td><td>M1A1</td></tr>
<tr><td colspan="3">M1 for $2b - 5a = 12$ is implied by an incorrect value for a with a correct ft answer for b
eg $a = 8$ $b = 26$</td><td>M1A0A1ft</td></tr>
<tr><td colspan="3">An incorrect but exact value for a seen in working (eg $\frac{8}{3}$) with a rounded value for a on answer line (eg 2.6)
Allow ft for b from the exact or the rounded value</td><td></td></tr>
<tr><td colspan="3">$a = 4$ and -4 with one or both of $b = 16$ and -4</td><td>M1A0A1ft</td></tr>
<tr><td colspan="3">$a = 4$ and -4 (no values for b or incorrect values for b)</td><td>M1A0A0ft</td></tr>
</table>

Q	Answer	Mark	Comments
4	**Alternative method 1**		
	$y + 4x = c$ or $y = –4x + c$ or gradient = –4	M1	oe c can be any value other than 6 may be implied
	$1 + 4 \times 2 = c$ or $1 = (\text{their } –4) \times 2 + c$ or $c = 9$	M1	oe their –4 can only be 4 or $\frac{1}{4}$ implied by a correct equation of B eg $y – 1 = –4(x – 2)$ or $y + 4x = 9$ or $y = –4x + 9$
	$2d + 4d = \text{their } 9$ or $2d = (\text{their } –4)d + \text{their } 9$ or $6d = 9$ or $9 \div 6$	M1dep	oe substitution of $(d, 2d)$ into their equation of B equation with no algebraic denominator dep on 2nd M1
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	oe eg $\frac{9}{6}$
	Alternative method 2		
	$y + 4x = c$ or $y = –4x + c$ or gradient = –4	M1	oe c can be any value other than 6 may be implied
	$\frac{2d-1}{d-2} = \text{their } –4$	M1	oe their –4 can only be 4 or $\frac{1}{4}$ may be implied
	$2d – 1 = \text{their } –4(d – 2)$ or $6d = 9$ or $9 \div 6$	M1dep	oe equation with no algebraic denominator dep on 2nd M1
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	oe eg $\frac{9}{6}$

Additional Guidance is on the next page

4 cont	**Additional Guidance**	
	Ignore simplification or conversion if correct answer seen	
	Condone answer (1.5, 3) oe	
	gradient = $-4x$ must be recovered	
	3rd M1 Allow $(d, 2d)$ to be $(x, 2x)$ etc	
	3rd M1 Do not allow use of $(2d, d)$ to be a misread	
	A correct equation in d with no algebraic denominator implies M1M1M1 eg $2d - 1 = -4(d - 2)$ or $2d = -4d + 9$ or $6d = 9$	M1M1M1
	Alt 1 gradient = 4 $y = 4x - 7$ $2d = 4d - 7$ $d = 3.5$	M0 M1 M1A0
	Alt 1 gradient = $\frac{1}{4}$ $y = \frac{1}{4}x + \frac{1}{2}$ $2d = \frac{1}{4}d + \frac{1}{2}$ $d = \frac{2}{7}$	M0 M1 M1A0
	gradient -4 followed by correct method using gradient 4 or $\frac{1}{4}$ for 2nd and 3rd marks can score a maximum of M2 eg Alt 1 gradient -4 $1 = 4 \times 2 + c$ $2d = 4d - 7$	M0M1M1
	gradient -4 followed by correct method using gradient 4 or $\frac{1}{4}$ for 2nd mark (but not the 3rd mark) can score a maximum of M1 eg Alt 1 gradient -4 $y = \frac{1}{4}x + \frac{1}{2}$ (no further valid work)	M0M1M0

Q	Answer	Mark	Comments
5	–3 –2 –1 with no other values	B3	any order B2 –3 –2 –1 with one other value or any two of –3 –2 –1 with no other values or inequality for which the only integer values are –3 –2 –1 eg $-4 < x < 0$ or $-3 \leqslant x \leqslant -1$ or $-4 < x \leqslant -1$ B1 $-4 < x < 4$ or –3 –2 –1 (0) 1 2 3 with no other values or one of –3 –2 –1 with no other values or $x^2 < \frac{48}{3}$ or $x^2 < 16$ or $3(x + 4)(x - 4) < 0$ or $(x + 4)(x - 4) < 0$

Additional Guidance is on the next page

Additional Guidance is on the next page

5 cont	**Additional Guidance**	
	B1 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
	Answer –3 –2 –1 with no other values (no need to check working)	B3
	$-4 < x < 0$ is equivalent to the two inequalities $x > -4$ $x < 0$ etc	B2
	For B1 allow equivalent factorised inequalities or equivalent inequalities with coefficient 1 for x^2 eg1 $(3x + 12)(x - 4) < 0$ eg2 $3(4 + x)(4 - x) > 0$ eg3 $x^2 - \frac{48}{3} < 0$	 B1 B1 B1
	$(-4, 0)$ or $[-3, -1]$ etc	B2
	$(-4, 4)$	B1
	Only $x > -4$ or only $x < \pm 4$ or only $x < 4$	B0
	Condone B3 response in working with any inequality on answer line	B3
	Condone B3 response in working with 3 on answer line (3 is likely to be the number of integers)	B3
	Only invalid inequalities with no or incorrect answer	B0
	Only equations with no or incorrect answer	B0

Q	Answer	Mark	Comments
6	**Alternative method 1** Expands the given brackets		
	$((2n + 1)^2 =)$ $4n^2 + 2n + 2n + 1$ or $((2n - 1)^2 =)$ $4n^2 - 2n - 2n + 1$	M1	oe expansion eg $((2n + 1)^2 =)$ $4n^2 + 4n + 1$ may be seen in a grid may be seen embedded in second mark ignore any denominator
	$4n^2 + 2n + 2n + 1 - 4n^2 + 2n + 2n - 1$ or $4n^2 + 4n + 1 - 4n^2 + 4n - 1$ or $4n^2 + 2n + 2n + 1 - (4n^2 - 2n - 2n + 1)$ **and** $8n$ with no errors seen or $4n^2 + 4n + 1 - (4n^2 - 4n + 1)$ **and** $8n$ with no errors seen	M1dep	terms in any order ignore any denominator
	M2 seen and valid explanation	A1	eg1 M2 seen and $\frac{8n}{4} = 2n$ eg2 M2 seen and $8n$ is even and when divided by 4 it is even
	Alternative method 2 Difference of two squares		
	$(2n + 1 + 2n - 1)(2n + 1 - (2n - 1))$ or $(2n + 1 + 2n - 1)(2n + 1 - 2n + 1)$	M1	ignore any denominator
	M1 seen and $4n \times 2$ with no errors seen	M1dep	ignore any denominator
	M2 seen and valid explanation	A1	eg1 M2 seen and $\frac{4n\times 2}{4} = 2n$ eg2 M2 seen and $\frac{8n}{4} = 2n$ eg3 M2 seen and $8n$ is even and when divided by 4 it is even

Additional Guidance is on the next page

	Additional Guidance	
	Do not allow missing brackets even if recovered	
	Alt 1 $4n^2 + 4n + 1 – 4n^2 – 4n + 1$	M1M0
	Alt 1 $4n^2 + 2n + 2n + 1 – (4n^2 – 2n – 2n + 1)$ $= 4n^2 + 4n + 1 – 4n^2 – 4n – 1 = 8n$ (8n but error seen)	M1 M0
	Alt 1 Only $8n$	M0M0
	Alt 1 2nd M1 Allow unnecessary brackets eg $4n^2 + 4n + 1 – (4n^2 – 4n + 1) = (4n^2 – 4n^2) + (4n + 4n) + (1 – 1)$	 M1M1
	Alt 2 $(2n + 1 + 2n – 1)(2n + 1 – 2n – 1)$	M0M0
6 cont	Alt 2 $(2n + 1 + 2n – 1)(2n + 1 – (2n – 1))$ $= (2n + 1 + 2n – 1)(2n + 1 – 2n – 1) = 4n \times 2$ ($4n \times 2$ but error seen)	M1 M0
	Alt 2 $(2n + 1 + 2n – 1)(2n + 1 – (2n – 1)) = 8n$	M1M0
	Alt 2 Only $4n \times 2$	M0M0
	Response only referring to odds and evens or only involving substitution	M0M0A0
	Assuming the expression simplifies to 2k and working back could score up to M1M1	
	Setting up an equation eg $(2n + 1)^2 – (2n – 1)^2 = 4$ could score up to M1M1	
	For A1 do not allow incorrect use of = eg $4n^2 + 2n + 2n + 1 – 4n^2 + 2n + 2n – 1$ $= \frac{8n}{4} = 2n$	 M1M1 A0

Q	Answer	Mark	Comments
7	240	B2	B1 $2 \times 5 \times 4 \times 3 \times 2$ or 2×120 or $2 \times 5!$ or 240 seen SC1 answer 120 or 360 or 480 or 600 or 720
	Additional Guidance		
	Ignore × 1 for B1		
	240 in working lines with 60 on answer line		B1
	720 in working lines with 1440 on answer line		Zero
	Allow dots for multiplication but do not allow addition		

Q	Answer	Mark	Comments
8(a)	$3x^2$ or $-10x$	M1	oe eg $3 \times x^{3-1}$ or $-2 \times 5x^1$
	$3x^2 - 10x - 4 = 0$ or $-3x^2 + 10x + 4 = 0$	A1	must show = 0
	Additional Guidance		
	M1 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts		
	Ignore extra terms eg $3x^2 - 10x + c$		M1
	$3x^2 - 10x = 4$ (even if $3x^2 - 10x - 4 = 0$ in (b))		M1A0
	$3x^2 - 10x - 4$ (even if $3x^2 - 10x - 4 = 0$ in (b))		M1A0
	$3x^2 - 10x - 4 = 0$ seen in working with $3x^2 - 10x - 4$ on answer line		M1A1
	Condone for M1 $y = 3x^2 \ldots$ etc (may still score A1 if recovered)		
	Answer $y = 3x^2 - 10x - 4 = 0$		M1A0

Q	Answer	Mark	Comments
8(b)	$\dfrac{--10 \pm \sqrt{(-10)^2 - 4 \times 3 \times -4}}{2 \times 3}$ or $\dfrac{10 \pm \sqrt{148}}{6}$ or $\dfrac{5}{3} \pm \sqrt{\dfrac{37}{9}}$ or two correct solutions with at least one not to 3 sf	M1	oe eg $\dfrac{5 \pm \sqrt{37}}{3}$ correct attempt to solve their $ax^2 + bx + c$ (= 0) from (a) a, b and c all non-zero eg 3.69(4…) **and** –0.36(09…) or 3.7 **and** –0.36(09…)
	3.69 and –0.361	A1ft	correct or ft any answers that have at least 4 sf must be rounded to 3 sf at least one answer must have at least 4 sf
	Additional Guidance		
	-10^2 used for $(-10)^2$ is M0 unless recovered		
	10^2 is equivalent to $(-10)^2$		
	Not using ± is M0 unless recovered		
	A short dividing line or a short square root symbol is M0 unless recovered		
	√((–10)^2 – 4 × 3 × –4) is correct for $\sqrt{(-10)^2 - 4 \times 3 \times -4}$		
	Correct factorisation of their $ax^2 + bx + c$ (= 0) from (a) scores at least M1		
	(a) $3x^2 - 10x + 4 = 0$ (b) $\dfrac{--10 \pm \sqrt{(-10)^2 - 4 \times 3 \times 4}}{2 \times 3}$ 2.87 and 0.465		M1A1ft
	(a) $3x^2 - 10x = 4$ (b) up to 2 marks can be scored if using $3x^2 - 10x - 4 = 0$		
	(a) $3x^2 - 10x - 8$ (b) up to 2 marks can be scored if using $3x^2 - 10x - 8 = 0$		
	One solution correct does not imply M1		
	Both solutions seen in working but only one on answer line		M1A0
	3.69 and –0.361 in working with –3.69 and 0.361 on answer line		M1A0

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="6">9(a)</td><td>$30 + 12k$ or $12k + 30$</td><td>B1</td><td>allow factorised eg $6(5 + 2k)$</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">$30 + 12k$ seen in working but incorrect answer eg $5 + 2k$ or $–2.5$</td><td>B0</td></tr>
<tr><td colspan="2">Answer line $30 + 12k$ and expression for the nth term eg $30 + 4nk – 4k$</td><td>B0</td></tr>
<tr><td colspan="2">$30 + 8k + 4k$</td><td>B0</td></tr>
<tr><td colspan="2">$30 + 12k$ unambiguously indicated as 4th term (eg in given sequence) with answer line blank</td><td>B1</td></tr>
</table>

Q	Answer	Mark	Comments
9(b)	**Alternative method 1** Works out a correct expression for the 100th term		
	$30 + 99 \times 4k$ or $30 + 396k$ or $100 \times 4k + 30 - 4k$	M1	oe eg $30 + (100 - 1) \times 4k$ or $30 + 4k + 98 \times 4k$ or $30 + 8k + 97 \times 4k$ or $30 + 12k + 96 \times 4k$
	$99 \times 4k = 525 - 30$ or $396k = 495$ or $495 \div 396$	M1dep	oe terms must be collected in an equation eg $396k - 495 = 0$
	1.25 or $\frac{5}{4}$ or $1\frac{1}{4}$	A1	oe eg $\frac{495}{396}$
	Alternative method 2 Uses a common difference (eg d)		
	$30 + 99 \times d$ or $30 + 99d$	M1	oe eg $30 + (100 - 1) \times d$
	$4k = \frac{525-30}{99}$ or $4k = \frac{495}{99}$ or $4k = 5$ or $5 \div 4$	M1dep	oe terms must be collected in an equation eg $4k - 5 = 0$
	1.25 or $\frac{5}{4}$ or $1\frac{1}{4}$	A1	oe eg $\frac{495}{396}$
	Alternative method 3 Uses their (a) to work out an expression for the 100th term		
	their (a) + $96 \times 4k$ or their (a) + $384k$	M1	their (a) must be in terms of k their (a) cannot be $30 + 4k$ or $30 + 8k$
	Collection of terms for their (a) + $384k = 525$	M1dep	their (a) must be of the form $c + dk$ $c \neq 0$ $d \neq 0$
	Solution to their equation rounded to 1 dp or better	A1ft	ft their (a) and M2

Additional Guidance is on the next page

9(b) cont	**Additional Guidance**	
	Ignore simplification or conversion if correct answer seen	
	Alt 1 Do not allow M1 if seen embedded eg in formula for S_n	
	Alt 3 (a) $12k$ (b) $12k + 384k$ $396k = 525$ 1.326	M1M0A0ft
	Alt 3 (a) $30 + 16k$ (b) $30 + 16k + 384k$ $400k = 525 – 30$ 1.238	M1M1A1ft
	Alt 3 (a) $12k + 60$ (b) $12k + 60 + 96 \times 4k$ $396k = 525 – 60$ 1.2	M1M1A1ft

Q	Answer	Mark	Comments
10	D	B1	

Q	Answer	Mark	Comments
11	$(0 <)\ x < 60$ or $(0 \leqslant)\ x < 60$	B2	B1 $\cos x > \frac{5-3}{4}$ or $\cos x > \frac{2}{4}$ or $\cos x > \frac{1}{2}$ or $x < \cos^{-1}\frac{1}{2}$ or $a < x < 60$ where a is a non-zero value less than 60 or $b \leqslant x < 60$ where b is a value less than 60 SC1 $(0 <)\ x \leqslant 60$ or $(0 \leqslant)\ x \leqslant 60$

Additional Guidance

Answer $(0 <)\ x < 60$ (can ignore working lines)	B2
$60 > x > 0$ is equivalent to $0 < x < 60$ etc	
$0 < x < 60$ is equivalent to the two inequalities $x > 0$ $x < 60$ etc	B2
Allow decimals for B1 responses eg $\cos x > 0.5$	B1
For B1 condone $\cos x => \frac{1}{2}$ for $\cos x > \frac{1}{2}$	
$\cos x > \frac{1}{2}$ followed by $x > \cos^{-1}\frac{1}{2}$	B1
Only $x > \cos^{-1}\frac{1}{2}$	B0
(0, 60)	B2
[0, 60)	B2
(0, 60]	SC1
[0, 60]	SC1

Q	Answer	Mark	Comments
12	$3x$ or $-2x^{-1}$ or $0.75x^{-2}$	M1	oe must have powers of x simplified eg $\frac{12x}{4}$ or $-\frac{2}{x}$ or $\frac{3}{4x^2}$
	$3x$ and $-2x^{-1}$ and $0.75x^{-2}$	M1dep	oe must have powers of x simplified eg $\frac{12x}{4}$ and $-\frac{2}{x}$ and $\frac{3}{4x^2}$
	Any one of $3x$ **and** $3(x^0)$ or $-2x^{-1}$ **and** $2x^{-2}$ or $0.75x^{-2}$ **and** $-1.5x^{-3}$	M1	oe eg $\frac{12x}{4}$ **and** $\frac{12}{4}x^{1-1}$ or $-\frac{2}{x}$ **and** $\frac{2}{x^2}$ or $-\frac{2}{x}$ **and** $--2x^{-2}$ or $\frac{3}{4x^2}$ **and** $-\frac{3}{2x^3}$ implies 1st M1 for the derivatives x may be (–1)
	At least two of $3x$ **and** $3(x^0)$ or $-2x^{-1}$ **and** $2x^{-2}$ or $0.75x^{-2}$ **and** $-1.5x^{-3}$	M1dep	oe dep on 3rd M1 for the derivatives x may be (–1)
	All three terms and their derivatives correct and 6.5	A1	oe eg all three terms and their derivatives correct and $\frac{13}{2}$ for the derivatives x may be (–1) SC3 104

Additional Guidance is on the next page

	Additional Guidance	
12 cont	Up to M4 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
	$\dfrac{3}{4x^2}$ seen but subsequently incorrectly simplified eg $12x^{-2}$ (subsequent marks may be scored)	(1st) M1
	Correct answer after correct use of quotient rule or product rule	M4A1
	Incorrect answer after use of quotient rule or product rule	Zero
	Condone $y = 3 + 2x^{-2}\ldots$ etc	
	All three terms and their derivatives correct and 6.5 in working but different answer eg $y = 6.5x\ \ldots$	M4A0
	SC3 is for multiplying the numerator by $4x^{-2}$ with no subsequent errors	

Q	Answer	Mark	Comments
13	**Alternative method 1**		
	$(x-1)^2+(y-9)^2=25$	B3	B2 $(x-1)^2+(y-9)^2=5^2$ or (1, 9) **and** radius = 5 or (1, 9) **and** radius^2 = 5^2 or (1, 9) **and** radius^2 = 25 B1 $(x-1)^2+(y-9)^2=k$ or $(x-\dots)^2+(y-\dots)^2=25$ or $(x-\dots)^2+(y-\dots)^2=5^2$ or (1, 9) or $\frac{-2+4}{2}$ oe **and** $\frac{5+13}{2}$ oe or radius = 5 or radius^2 = 5^2 or radius^2 = 25
	Alternative method 2 Uses perpendicular lines where (x, y) is a point on the circle		
	$\frac{y-5}{x--2}\times\frac{y-13}{x-4}=-1$	M1	oe eg $(y-5)(y-13)=-1(x+2)(x-4)$
	$y^2-18y+65+x^2-2x-8=0$	M1dep	oe equation of circle with brackets expanded and fractions eliminated eg $y^2-18y+65=-x^2+2x+8$
	$(x-1)^2+(y-9)^2=25$	A1	
	Additional Guidance		
	$a=1$ $b=9$ $c=25$ implies $(x-1)^2+(y-9)^2=25$		
	Alt 1 (1, 9) may be implied eg $x=1$ $y=9$ or 1, 9		B1
	Alt 1 $(x+3)^2+(y+4)^2=5^2$		B1
	Alt 1 $(x-1)^2+(y-9)^2=5$ (with no indication that radius = 5)		B1
	Alt 1 $r=5$		B1
	Alt 1 diameter = 10		B0

	$(x-1)^2 + (y-9)^2 = 25$ in working lines with brackets expanded on answer line	B2

Q	Answer	Mark	Comments
14	$4(x+15) + 4(x+15) - 40 = 180$ or $8(x+15) - 40 = 180$ or $4(x+15) = \frac{180+40}{2}$ or $4(x+15) - 40 = \frac{180-40}{2}$ or $y + 4(x+15) = 180$ **and** $y = 4(x+15) - 40$	M1	oe equation in x or pair of equations in x and y y may be any letter other than x eg $180 - (4x + 60) + 40 = 4x + 60$ or $4(x+15) = 110$ or $4(x+15) - 40 = 70$ or $y + 4x = 120$ **and** $y = 4x + 20$ implied by $y = 70$
	$4x + 60 + 4x + 60 - 40 = 180$ or $8x + 120 - 40 = 180$ or $8x = 100$ or $100 \div 8$ or $4x = 50$ or $50 \div 4$	M1dep	oe equation or calculation equation with brackets expanded and fractions eliminated eg $120 - 4x + 40 = 4x + 60$ or $8x + 80 = 180$ or $4x + 60 = 110$ or $4x + 20 = 70$
	12.5 or $\frac{25}{2}$ or $12\frac{1}{2}$	A1	oe eg $\frac{100}{8}$ or $\frac{50}{4}$ SC2 2.5 oe
	Additional Guidance		
	Ignore simplification or conversion if correct answer seen		
	2nd M1 Allow unnecessary brackets eg $(4x + 60) + (4x + 60) - 40 = 180$		M1M1
	1st M1 may be implied if expansion error seen eg $4(x+15) = 4x + 15$ (may be seen on diagram) $4x + 15 + 4x + 15 - 40 = 180$		M1M0
	Only $4x + 15 + 4x + 15 - 40 = 180$		M0
	SC2 is when they have angle *PQR* 40° larger than angle *PSR*		

Q	Answer	Mark	Comments
15	**Alternative method 1** Processes the brackets then divides		
	$\frac{5x}{10} + \frac{6x}{10}$	M1	oe valid common denominator with both numerators correct eg $\frac{10x}{20} + \frac{12x}{20}$
	$\frac{11x}{10}$	A1	oe single term eg $\frac{22x}{20}$ or $1.1x$ may be implied eg by single term with roots evaluated that is equivalent to $\frac{11}{5x^2}$
	$\frac{x^{6\div2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied eg by multiplication by $\frac{2}{x^3}$
	their $\frac{11x}{10} \times \frac{2}{x^3}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe multiplication eg $\frac{11x}{10} \times 2x^{-3}$ their $\frac{11x}{10}$ can be unprocessed dep on 2nd M1
	$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$

Mark scheme and Additional Guidance continues on the next page

15 cont	**Alternative method 2** Divides then expands the brackets		
	$\frac{x^{6\div 2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied eg by multiplication by $\frac{2}{x^3}$
	$\left(\frac{x}{2}+\frac{3x}{5}\right) \times \frac{2}{x^3}$	M1dep	oe multiplication eg $\left(\frac{x}{2}+\frac{3x}{5}\right) \times 2x^{-3}$
	$\frac{2x}{2x^3} + \frac{6x}{5x^3}$ or $\frac{1}{x^2} + \frac{6}{5x^2}$	M1dep	oe expansion of brackets
	$\frac{10x}{10x^3} + \frac{12x}{10x^3}$ or $\frac{5}{5x^2} + \frac{6}{5x^2}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe valid common denominator with both numerators correct eg $\frac{10x^4}{10x^6} + \frac{12x^4}{10x^6}$ or $\frac{22x^4}{10x^6}$ roots must be processed
	$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$
	Additional Guidance		
	Any single fraction with roots evaluated that is equivalent to $\frac{11}{5x^2}$		4 marks
	Allow inclusion of ± from the square root for up to 4 marks		
	$\frac{11}{5x^2}$ in working with answer $\frac{11}{5}x^2$		4 marks
	Alt 1 $\frac{11x}{10}$ subsequently squared and not recovered		M1A1 M0M0A0

Q	Answer	Mark	Comments
16	**Alternative method 1** Uses $\frac{1}{2}ab\sin C$		
	$\frac{1}{2} \times 16 \times 16 \times \sin x$ or $128 \sin x$	M1	oe eg $\frac{1}{2} \times 16 \times 16 \times \sin(180 - 2y)$ x can be any letter or expression may be implied
	$\sin x = 120 \div \left(\frac{1}{2} \times 16 \times 16\right)$ or $\sin x = \frac{15}{16}$ or $\sin^{-1}\frac{15}{16}$ or $\sin^{-1}[0.93, 0.94]$ or [68.4, 70.12313]	M1dep	oe eg $\sin x = \frac{240}{256}$ or $\sin x$ = [0.93, 0.94] equation must have $\sin x =$ x can be any letter or expression
	$\frac{180 - \text{their}[68.4, 70.12313]}{2}$	M1dep	oe
	[54.93, 55.8]	A1	SC2 [75.82, 76.4]
	Alternative method 2 Works out perpendicular height		
	$120 \div \left(\frac{1}{2} \times 16\right)$ or $120 \div 8$ or 15	M1	
	$\cos x = \frac{\sqrt{16^2 - (\text{their } 15)^2}}{16}$ or $\cos^{-1}\frac{\sqrt{31}}{16}$ or $\tan x = \frac{15}{\sqrt{16^2 - (\text{their } 15)^2}}$ or $\tan^{-1}\frac{15}{\sqrt{31}}$ or [68.4, 70.12313]	M1dep	oe eg $\sin x = \frac{15}{16}$ or $\sin x$ = [0.93, 0.94] or $\cos x$ = [0.34, 0.35] or $\tan x$ = [2.69, 2.7] x can be any letter or expression
	$\frac{180 - \text{their}[68.4, 70.12313]}{2}$	M1dep	oe
	[54.93, 55.8]	A1	SC2 [75.82, 76.4]

Mark scheme and Additional Guidance continue on the next page

16 cont	**Alternative method 3** Works out perpendicular height		
	$120 \div \left(\frac{1}{2} \times 16\right)$ or $120 \div 8$ or 15	M1	oe
	$16 - \sqrt{16^2 - (\text{their } 15)^2}$ or $16 - \sqrt{31}$ or [10.4, 10.44]	M1dep	oe eg $\tan y = \frac{15}{16 - \sqrt{16^2 - (\text{their } 15)^2}}$ y can be any letter or expression
	$\tan^{-1} \frac{15}{\text{their } [10.4, 10.44]}$	M1dep	oe eg $\tan^{-1}$ [1.43, 1.44231]
	[54.93, 55.8]	A1	SC2 [75.82, 76.4]
	Additional Guidance		
	Alt 1 y = [68.4, 70.12313]		M1M1
	Condone sin = for sin x = etc Condone $\sin^{-1} = 0.9375$ for $\sin^{-1} 0.9375$ etc		
	SC2 is for omitting the 0.5 from the area of triangle formula		
	After scoring M1M1, the 3rd M1 is for any full method eg Alt 1 68.6 Cosine rule used to work out the third side of the triangle followed by sine rule to work out y (up to $\sin^{-1}$…) If there are no errors seen in the method the 3rd M1 is awarded and possibly the A1 as well		M1M1

Q	Answer	Mark	Comments
17	Elimination of one variable making an equation with at least two terms correct	M1	eg1 (elimination of b by adding 1st and 2nd equations) $5a + 3c = -1$ with at least two terms correct eg2 (elimination of a by doubling 1st equation and subtracting 3rd equation) $5b - 7c = -1$ with at least two terms correct

	Elimination of one variable making an equation with at least two terms correct and elimination of the same variable making a different equation with at least two terms correct	M1dep	eg1 (elimination of b by adding 1st and 2nd equations and elimination of b by trebling 3rd equation and subtracting 1st equation) $5a + 3c = –1$ with at least two terms correct and $5a + 11c = 23$ with at least two terms correct eg2 (elimination of a by doubling 1st equation and subtracting 3rd equation and elimination of a by doubling 3rd equation and subtracting 2nd equation) $5b – 7c = –1$ with at least two terms correct and $5b + c = 23$ with at least two terms correct
	Correct equation in one variable with two correct equations in the same two variables	M1dep	eg $3c – 11c = –1 – 23$ or $–8c = –24$ or $c = 3$ with $5a + 3c = –1$ and $5a + 11c = 23$
	Two correct values with two correct equations in the same two variables	A1	eg $c = 3$ and $a = –2$ with $5a + 3c = –1$ and $5a + 11c = 23$
	$a = –2$ $b = 4$ $c = 3$ with two correct equations in the same two variables	A1	eg $a = –2$ $b = 4$ $c = 3$ with $5a + 3c = –1$ and $5a + 11c = 23$

Additional Guidance is on the next page

<table>
<tr><td rowspan="11">17 cont</td><td colspan="2">Additional Guidance</td></tr>
<tr><td>The two correct equations in the same two variables referred to in the scheme are a pair from one of these columns
<table><tr><td>$15b - 13c = 21$</td><td>$5a + 3c = -1$</td><td>$13a + 9b = 10$</td></tr><tr><td>$5b - 7c = -1$</td><td>$5a + 11c = 23$</td><td>$7a + 11b = 30$</td></tr><tr><td>$5b + c = 23$</td><td>$10a + 14c = 22$</td><td>$2a - 14b = -60$</td></tr></table>All equations have equivalents
eg equivalents for $5a + 3c = -1$ include $-5a - 3c = 1$ and $5a = -1 - 3c$</td><td></td></tr>
<tr><td>All equations in two variables must have terms collected
eg $a + 4a - 2c + 5c = 4 - 5$ requires simplification to $5a + 3c = -1$</td><td></td></tr>
<tr><td>$0a + 15b - 13c = 21$ is equivalent to $15b - 13c = 21$ etc</td><td></td></tr>
<tr><td>Equations with two terms correct include
eg1 (For $5b + c = 23$) $5b + c = 10$ and $-5b - c = 2$ and $5b - 3c = 23$
eg2 (For $5a + 3c = -1$) $5a + 6c = -1$ and $-5a - 3c = 4$ and $5a = 2 - 3c$</td><td></td></tr>
<tr><td>For equations with two terms correct the signs can be ignored if the modulus of the numbers in the correct equation are unchanged
eg For the correct equation $5b - 7c = -1$ (so modulus 5, 7 and 1)
equations with two terms correct include
$5b + 7c = 1$ and $5b - 7c = 1$ and $-5b - 7c = 1$ and $-5b - 7c + 1 = 0$</td><td></td></tr>
<tr><td>Up to M3 may be awarded for correct work with no, or incorrect answer,
even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td>Elimination of variables may be seen from other approaches
eg rearranges 1st equation to $a = 4 - 3b + 2c$ and substitutes into the 2nd and 3rd equations</td><td></td></tr>
<tr><td>Correct values with no working</td><td>Zero</td></tr>
<tr><td>Matrix method involving row reduction is equivalent to the methods in the mark scheme</td><td></td></tr>
<tr><td>Correct inverse matrix seen with three correct solutions</td><td>M3A2</td></tr>
</table>

Q	Answer	Mark	Comments
18	$\frac{40}{3+7} \times 7$ or 28	M1	oe eg $40 - \frac{40}{3+7} \times 3$ or $40 - 12$ may be seen on diagram may be implied
	20^2 + their 28^2 or $400 + 784$ or 1184 or $4\sqrt{74}$ or [34.4, 34.41]	M1	oe eg $\sqrt{20^2 + \text{their}28^2}$ or $\sqrt{1184}$ their 28 must be < 40 may be seen on diagram
	$40^2 + 9^2$ or $1600 + 81$ or 1681 or 41	M1	oe eg $\sqrt{40^2 + 9^2}$ or $\sqrt{1681}$ may be seen on diagram
	their 1681 = 25^2 + their 1184 $- 2 \times 25 \times \sqrt{\text{their}1184} \times \cos x$	M1dep	oe eg $\cos^{-1} \frac{25^2 + \text{their}1184 - \text{their}1681}{2 \times 25 \times \sqrt{\text{their}1184}}$ or $\cos^{-1}$ [0.07, 0.07442] dep on 2nd and 3rd M1 x may be *APC* or *A* etc
	[85.7, 86]	A1	

Additional Guidance is on the next page

18 cont	**Additional Guidance**	
	Up to M4 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
	If their *PG* is 28 do not allow use of a value other than 28 in subsequent working	
	$\frac{40}{3+7} \times 3 = 12$ $20^2 + 12^2 = 544$ $40^2 + 9^2 = 1681$ $\cos^{-1} \frac{25^2 + 544 - 1681}{2 \times 25 \times \sqrt{544}}$	M0 M1 M1 M1A0
	4th M1 Condone $\cos^{-1} = 0.07$ for $\cos^{-1} 0.07$ etc	
	4th M1 oes must be a fully correct method eg Uses cosine rule to work out angle *PCA* then uses sine rule to work out angle *APC* Must get to correct sine rule equation with no errors in method	
	Missing brackets must be recovered eg 4th M1 Do not allow $4\sqrt{74}^2$ unless recovered in subsequent working	
	When *AP* is used it must be 25	

Q	Answer	Mark	Comments
19	**Alternative method 1** Expands $(3x + 4)(2x – 3)$ first		
	$6x^2 – 9x + 8x – 12$ or $6x^2 – x – 12$	M1	oe 4 terms with at least 3 correct implied by $6x^2 – x + k$ or $px^2 – x – 12$ where k and p are non-zero constants may be seen in a grid
	$30x^3 – 45x^2 + 40x^2 – 60x – 12x^2$ $+ 18x – 16x + 24$ or $30x^3 – 5x^2 – 60x – 12x^2 + 2x + 24$	M1	oe full expansion with correct multiplication of their 3 or 4 terms by $5x$ or $–2$ may be seen in a grid
	$30x^3 – 17x^2 – 58x + 24$	A1	terms in any order
	Alternative method 2 Expands $(2x – 3)(5x – 2)$ first		
	$10x^2 – 4x – 15x + 6$ or $10x^2 – 19x + 6$	M1	oe 4 terms with at least 3 correct implied by $10x^2 – 19x + k$ or $px^2 – 19x + 6$ where k and p are non-zero constants may be seen in a grid
	$30x^3 – 12x^2 – 45x^2 + 18x + 40x^2$ $– 16x – 60x + 24$ or $30x^3 – 57x^2 + 18x + 40x^2 – 76x$ $+ 24$	M1	oe full expansion with correct multiplication of their 3 or 4 terms by $3x$ or 4 may be seen in a grid
	$30x^3 – 17x^2 – 58x + 24$	A1	terms in any order

Mark scheme and Additional Guidance continues on the next page

<table>
<tr><td rowspan="15">19 cont</td><td colspan="3">Alternative method 3 Expands $(3x + 4)(5x – 2)$ first</td></tr>
<tr><td>$15x^2 – 6x + 20x – 8$
or
$15x^2 + 14x – 8$</td><td>M1</td><td>oe
4 terms with at least 3 correct
implied by $15x^2 + 14x + k$
or $px^2 + 14x – 8$
where k and p are non-zero constants
may be seen in a grid</td></tr>
<tr><td>$30x^3 – 12x^2 + 40x^2 – 16x – 45x^2$
$+ 18x – 60x + 24$
or
$30x^3 + 28x^2 – 16x – 45x^2 – 42x$
$+ 24$</td><td>M1</td><td>oe
full expansion with correct multiplication of their 3 or 4 terms by $2x$ or $–3$
may be seen in a grid</td></tr>
<tr><td>$30x^3 – 17x^2 – 58x + 24$</td><td>A1</td><td>terms in any order</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">For terms seen in a grid accept $8x$ for $+8x$ etc</td><td></td></tr>
<tr><td colspan="2">2nd M1
A full expansion will be 8 terms if 4 terms are used in first expansion
A full expansion will be 6 terms if 3 terms are used in first expansion</td><td></td></tr>
<tr><td colspan="2">Alt 1
$6x^2 + 9x – 8x – 12$ only 2 terms correct
$(6x^2 + 9x – 8x – 12)(5x – 2)$
$= 30x^3 + 45x^2 – 40x^2 – 60x – 12x^2 + 18x – 16x + 24$
8 terms with correct multiplication of their 4 terms by $5x$</td><td>M0

M1A0</td></tr>
<tr><td colspan="2">Alt 2
$10x^2 – 19x – 5$ implied 4 terms with 3 correct
$(3x + 4)(10x^2 – 19x – 5) = 30x^3 – 54x^2 – 15x + 40x^2 – 76x – 20$
6 terms with correct multiplication of their 3 terms by 4</td><td>M1
M1A0</td></tr>
<tr><td colspan="2">1st M1 with a 4-term expansion followed by incorrect simplification to 3 terms can still score the 2nd M1 using their 3 terms</td><td></td></tr>
<tr><td colspan="2">One single expansion is full marks or zero</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
20(a)	Shows substitution of $x = \frac{1}{2}$	M1	eg $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} - 9$ or $2 \times \frac{1}{8} + 11 \times \frac{1}{4} + 12 \times \frac{1}{2} - 9$ or $\frac{1}{4} + \frac{11}{4} + 6 - 9$
	Shows substitution of $x = \frac{1}{2}$ and evaluates to zero	A1	eg $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} - 9 = 0$ or $2 \times \frac{1}{8} + 11 \times \frac{1}{4} + 12 \times \frac{1}{2} - 9 = 0$ or $\frac{1}{4} + \frac{11}{4} + 6 - 9 = 0$

Additional Guidance is on the next page

	Additional Guidance	
20(a) cont	Allow use of 0.5 and/or absence of multiplication signs eg1 $2(0.5)^3 + 11(0.5)^2 + 12(0.5) – 9 = 0$ eg2 $2\left(\frac{1}{8}\right) + 11\left(\frac{1}{4}\right) + 12\left(\frac{1}{2}\right) – 9$	 M1A1 M1A0
	Allow working in stages eg $2(0.5)^3 + 11(0.5)^2 + 12(0.5) = 9 \quad 9 – 9 = 0$	 M1A1
	Condone incorrect use of = eg $2(0.5)^3 + 11(0.5)^2 + 12(0.5) = 9 – 9 = 0$	 M1A1
	Condone $2 \times \frac{1}{2}^3$ or $2 \times \left(\frac{1^3}{2}\right)$ etc	
	Ignore algebraic division or other substitution attempts	
	Only stating $f\left(\frac{1}{2}\right)$ or only stating $f\left(\frac{1}{2}\right) = 0$	M0A0
	Calculation error(s) will be A0 eg1 $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} – 9 = \frac{1}{8} + \frac{11}{4} + 6 – 9 = 0$ eg2 $\frac{1}{4} + \frac{11}{4} + 6 – 9 = 4 + 6 – 9 = 0$	 M1A0 M1A0
	May be seen as synthetic division eg $\begin{array}{c\|c\|c\|c\|c} & 2 & 11 & 12 & –9 \\ 0.5 & & 1 & 6 & 9 \\ \hline & 2 & 12 & 18 & 0 \end{array}$ (with the bottom right entry blank award M1A0) (with an error award M0A0)	M1A1

Q	Answer	Mark	Comments
20(b)	**Alternative method 1**		
	$x^2 + 6x$ … or $2 \times (-3)^3 + 11 \times (-3)^2 + 12 \times (-3) - 9$	M1	oe eg $2x-1\overline{)2x^3+11x^2+12x-9}$ with quotient $x^2 + 6x$ … or $(2x - 1)(x^2 + bx + c)$ and $b = 6$ or $2 \times -27 + 11 \times 9 + 12 \times -3 - 9$ or $-54 + 99 - 36 - 9$
	$x^2 + 6x + 9$ **or** $(x + 3)(x + 3)$ **or** $(x + 3)^2$	M1dep	oe eg $2x-1\overline{)2x^3+11x^2+12x-9}$ with quotient $x^2 + 6x + 9$ or $(2x - 1)(x^2 + bx + c)$ and $b = 6$ and $c = 9$
	$x^2 + 6x + 9$ **and** $(x + 3)(x + 3)$ or $x^2 + 6x + 9$ **and** $\dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$ or $x^2 + 6x + 9$ **and** $6^2 - 4 \times 1 \times 9 = 0$ or $(2x - 1)(x + 3)(x + 3)$	M1dep	oe eg $x^2 + 6x + 9$ **and** $(x + 3)^2$ or $x^2 + 6x + 9$ **and** $\dfrac{-6}{2}$ or $x^2 + 6x + 9$ **and** $36 - 36 = 0$ or $(2x - 1)(x + 3)^2$
	M3 **and** indication that there are exactly two solutions	A1	eg1 $x^2 + 6x + 9$ **and** $(x + 3)(x + 3)$ **and** 0.5 **and** –3 eg2 $x^2 + 6x + 9$ **and** $\dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$ **and** 0.5 **and** –3 eg3 $(2x - 1)(x + 3)(x + 3)$ **and** repeated bracket so exactly two solutions/roots/answers/factors

Mark scheme and Additional Guidance continue on the next four pages

20(b) cont	**Alternative method 2**		
	$6x^2 + 22x + 12 = 0$ or $(6x + 4)(x + 3) = 0$ or $\dfrac{-22 \pm \sqrt{22^2 - 4 \times 6 \times 12}}{2 \times 6}$ or $\dfrac{-22 \pm \sqrt{196}}{12}$	M1	condone omission of = 0 oe eg $(2x + 6)(3x + 2) = 0$ or $2(x + 3)(3x + 2) = 0$ or $-\dfrac{11}{6} \pm \sqrt{-2 + \dfrac{121}{36}}$ or $-\dfrac{11}{6} \pm \sqrt{\dfrac{49}{36}}$
	$x = -\dfrac{2}{3}$ **and** $x = -3$	M1dep	allow [–0.67, –0.66] for $-\dfrac{2}{3}$
	$x = -\dfrac{2}{3}$ **and** (–3, 0)	M1dep	allow [–0.67, –0.66] for $-\dfrac{2}{3}$ ignore y-coordinate for $x = -\dfrac{2}{3}$ (–3, 0) may be seen on a graph
	M3 **and** indication that there are exactly two solutions	A1	eg $x = -\dfrac{2}{3}$ **and** (–3, 0) **and** a turning point on the x-axis so two solutions/roots

Mark scheme and Additional Guidance continue on the next three pages

20(b) cont	**Alternative method 3**		
	Sketch of cubic graph with maximum turning point at (–3, 0)	M1	condone minimum turning point at (–3, 0)
	Sketch of cubic graph with maximum turning point at (–3, 0) **and** minimum turning point in the third quadrant	M1dep	
	Sketch of cubic graph with maximum turning point at (–3, 0) **and** minimum turning point in the third quadrant **and** intersecting the positive *x*-axis at $\frac{1}{2}$	M1dep	–3 and $\frac{1}{2}$ must both be correctly labelled on the *x*-axis
	M3 **and** indication that there are exactly two solutions	A1	eg M3 **and** 0.5 **and** –3

Additional Guidance is on the next two pages

<table>
<tr><th rowspan="13">20(b) cont</th><th colspan="2">Additional Guidance</th></tr>
<tr><td>Up to M3 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td>Alt 1 Up to the first two marks may be seen in a grid
eg
<table>
<tr><td></td><td>x^2</td><td>$+6x$</td><td>$+9$</td></tr>
<tr><td>$2x$</td><td>$2x^3$</td><td>$12x^2$</td><td>$18x$</td></tr>
<tr><td>$–1$</td><td>$– x^2$</td><td>$–6x$</td><td>$– 9$</td></tr>
</table>
Condone missing + symbols in top row unless subsequently contradicted</td><td>M1M1</td></tr>
<tr><td>Alt 1 $x^2 + 6x + 9$ or $(x + 3)(x + 3)$ or $(x + 3)^2$</td><td>M1M1</td></tr>
<tr><td>Alt 1 $(2x – 1)(x + 3)(x + 3)$ or $(2x – 1)(x + 3)^2$</td><td>M1M1M1</td></tr>
<tr><td>Alt 1 $(2x – 1)(x + 3)(x + 3)$ with solutions 0.5 and –3</td><td>M1M1M1A1</td></tr>
<tr><td>Alt 1 $2x^2 + 5x – 3 = (2x – 1)(x + 3)$ 0.5 and –3</td><td>Zero</td></tr>
<tr><td>Alt 1 Examples of acceptable indications that there are exactly two solutions
eg1 $x = 0.5, –3, –3$ (Only) two solutions
eg2 $x = 0.5, –3, –3$ One root is a repeat
eg3 $(2x – 1)$ gives one solution $(x + 3)(x + 3)$ gives one solution
eg4 $(2x – 1)(x + 3)(x + 3)$ Two factors (only)</td><td></td></tr>
<tr><td>Alt 1 These are not acceptable indications that there are exactly two solutions
eg1 $(2x – 1)(x + 3)(x + 3)$ 3 and 0.5
eg2 $(x + 3)(x + 3)$ Exactly two solutions</td><td></td></tr>
<tr><td>Alt 1 Ignore other substitution attempts if using factor theorem for 1st M1</td><td></td></tr>
<tr><td>Alt 1 Allow absence of multiplication signs in factor theorem
eg $2(–3)^3 + 11(–3)^2 + 12(–3) – 9$</td><td>M1</td></tr>
<tr><td>Alt 1 Condone incorrect use of =
eg $2(–3)^3 + 11(–3)^2 + 12(–3) = 9 – 9$</td><td>M1</td></tr>
</table>

Additional Guidance continues on the next page

<table>
<tr><td rowspan="5">20(b) cont</td><td>Alt 1 Allow working in stages
eg $2(–3)^3 + 11(–3)^2 + 12(–3) = 9$ $9 – 9 = 0$</td><td>M1</td></tr>
<tr><td>Alt 1 Only stating f(–3) or only stating f(–3) = 0</td><td>M0</td></tr>
<tr><td>Alt 1 May be seen as synthetic division
eg
$$\begin{array}{r|c|c|c|c} & 2 & 11 & 12 & –9 \\ –3 & & –6 & –15 & 9 \\ \hline & 2 & 5 & –3 & 0 \end{array}$$</td><td>M1</td></tr>
<tr><td>Working in (a) eg algebraic division that is not used in (b) cannot score in (b)
eg (a) $$\begin{array}{r} x^2 + 6x + 9 \\ 2x-1\overline{)2x^3+11x^2+12x-9} \end{array}$$
(b) Not attempted</td><td>M0</td></tr>
<tr><td>Working in (a) eg algebraic division that is used in (b) can score in (b)
eg (a) $(2x – 1)(x^2 + 6x + 9)$
(b) Student shows an arrow from their working in (a)</td><td>M1M1</td></tr>
</table>

Q	Answer	Mark	Comments
21	**Alternative method 1**		
	$\tan x = \sqrt{\frac{3}{2}}$ or $\tan x = \frac{\sqrt{6}}{2}$ or [50.7, 50.8] or 51 or [230.7, 230.8] or 231	M1	oe eg $\tan^{-1}\sqrt{\frac{3}{2}}$ allow [1.22, 1.225] for $\sqrt{\frac{3}{2}}$
	$\tan x = -\sqrt{\frac{3}{2}}$ or $\tan x = -\frac{\sqrt{6}}{2}$ or [–50.8, –50.7] or –51 or [129.2, 129.3] or 129 or [309.2, 309.3] or 309	M1	oe eg $\tan^{-1} -\sqrt{\frac{3}{2}}$ allow [–1.225, –1.22] for $-\sqrt{\frac{3}{2}}$
	50.8 and 129.2 and 230.8 and 309.2 with no other angles in range [0, 360]	A2	A1 [50.7, 50.8] or 51 **and** [230.7, 230.8] or 231 or [129.2, 129.3] or 129 **and** [309.2, 309.3] or 309
	Alternative method 2		
	$\sin x = \sqrt{\frac{3}{5}}$ or $\sin x = \frac{\sqrt{15}}{5}$ or [50.7, 50.8] or 51 [129.2, 129.3] or 129	M1	oe eg $\sin^{-1}\sqrt{\frac{3}{5}}$ allow [0.77, 0.775] for $\sqrt{\frac{3}{5}}$
	$\sin x = -\sqrt{\frac{3}{5}}$ or $\sin x = -\frac{\sqrt{15}}{5}$ or [–50.8, –50.7] or –51 [230.7, 230.8] or 231 or [309.2, 309.3] or 309	M1	oe eg $\sin^{-1} -\sqrt{\frac{3}{5}}$ allow [–0.775, –0.77] for $-\sqrt{\frac{3}{5}}$
	50.8 and 129.2 and 230.8 and 309.2 with no other angles in range [0, 360]	A2	A1 [50.7, 50.8] or 51 **and** [129.2, 129.3] or 129 or [230.7, 230.8] or 231 **and** [309.2, 309.3] or 309

Mark scheme and Additional Guidance continues on the next 2 pages

21 cont	**Alternative method 3**		
	$\cos x = \sqrt{\frac{2}{5}}$ or $\cos x = \frac{\sqrt{10}}{5}$ or [50.7, 50.8] or 51 or [309.2, 309.3] or 309	M1	oe eg $\cos^{-1}\sqrt{\frac{2}{5}}$ allow [0.63, 0.6325] for $\sqrt{\frac{2}{5}}$
	$\cos x = -\sqrt{\frac{2}{5}}$ or $\cos x = -\frac{\sqrt{10}}{5}$ or [129.2, 129.3] or 129 or [230.7, 230.8] or 231	M1	oe eg $\cos^{-1}-\sqrt{\frac{2}{5}}$ allow [–0.6325, –0.63] for $-\sqrt{\frac{2}{5}}$
	50.8 and 129.2 and 230.8 and 309.2 with no other angles in range [0, 360]	A2	A1 [50.7, 50.8] or 51 **and** [309.2, 309.3] or 309 or [129.2, 129.3] or 129 **and** [230.7, 230.8] or 231

Additional Guidance is on the next page

	Additional Guidance	
21 cont	Allow t for tan x etc	
	tan $x = \pm\sqrt{\frac{3}{2}}$	M1M1
	Ignore any solutions outside the range [0, 360]	
	All four solutions with extra solutions in range [0, 360] scores M1M1A1 eg 50.8 and 230.8 and 129.2 and 309.2 and 180 and 60	 M1M1A1
	For A1 there may be extra solutions in range eg1 50.77 and 230.8 and 180 eg2 50.8 and 230.8 and 129.2 and 90	 M1M0A1 M1M1A1
	If answer line is blank, award any marks gained in the working lines	
	If correct angles are found in the working lines but only some are listed on the answer line award any M marks gained from the working lines award any A marks gained from the answer line eg1 working lines tan $x = \pm\sqrt{\frac{3}{2}}$ 50.8 230.8 129.2 309.2 answer line 50.76 230.76 129.2 eg2 working lines tan $x = \sqrt{\frac{3}{2}}$ 50.8 230.8 answer line 50.8 eg3 working lines tan $x = \sqrt{\frac{3}{2}}$ 50.8 230.8 tan $x = -\sqrt{\frac{3}{2}}$ 129.2 answer line 129.2	 M1M1 A1 M1M0A0 M1M1 A0
	Answers only (with no extra solutions in range) can score up to 4 marks 4 correct → 4 marks 3 correct → 3 marks 2 correct → 2 marks 1 correct → 1 mark	
	M1M0A1 or M0M1A1 are possible eg1 tan $x = \sqrt{\frac{3}{2}}$ 50.76 230.8 eg2 tan $x = -\sqrt{\frac{3}{2}}$ 129.2 and 309.2	 M1M0A1 M0M1A1
	Embedded answers can score up to M1M1A1	

Q	Answer	Mark	Comments
22	**Alternative method 1** Uses powers of 2		
	$(16^x =)\ 2^{4x}$ or $((16^x)^x =)\ (2^4)^{x^2}$	M1	implied by $((16^x)^x =)\ 2^{4x^2}$ may be implied by 3rd M1
	$((16^x)^x =)\ 2^{4x^2}$	M1dep	implied by $2^{4x^2 + 3x}$ may be implied by 3rd M1
	Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $4x^2 = –3x$ or $4x^2 + 3x = 0$ or $4x = –3$ or $2^{4x^2} = 2^{-3x}$ or $2^{4x^2 + 3x} = 2^0$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
	M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Mark scheme and Additional Guidance continues on the next 3 pages

22 cont	**Alternative method 2** Uses powers of 16		
	$((16^x)^x =)\ 16^{x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{\left(16^{\frac{1}{4}}\right)^{3x}}$	M1	implied by $\left(\frac{1}{2^{3x}} =\right) \frac{1}{16^{\frac{3x}{4}}}$ or $\left(\frac{1}{2^{3x}} =\right) 16^{-\frac{3x}{4}}$ may be implied by 3rd M1
	$((16^x)^x =)\ 16^{x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) \frac{1}{16^{\frac{3x}{4}}}$	M1dep	oe eg $((16^x)^x =)\ 16^{x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) 16^{-\frac{3x}{4}}$ may be implied by 3rd M1
	Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $x^2 = -\frac{3}{4}x$ or $4x^2 + 3x = 0$ or $16^{x^2} = 16^{-\frac{3x}{4}}$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
	M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Mark scheme and Additional Guidance continues on the next 2 pages

22 cont	**Alternative method 3** Uses powers of 4		
	$(16^x =)\ 4^{2x}$ or $((16^x)^x =)\ (4^2)^{x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{\left(4^{\frac{1}{2}}\right)^{3x}}$	M1	implied by $((16^x)^x =)\ 4^{2x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{4^{\frac{3x}{2}}}$ or $\left(\frac{1}{2^{3x}} =\right) 4^{-\frac{3}{2}x}$ may be implied by 3rd M1
	$((16^x)^x =)\ 4^{2x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) \frac{1}{4^{\frac{3x}{2}}}$	M1dep	oe $((16^x)^x =)\ 4^{2x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) 4^{-\frac{3}{2}x}$ may be implied by 3rd M1
	Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $2x^2 = -\frac{3}{2}x$ or $4x^2 + 3x = 0$ or $4^{2x^2} = 4^{-\frac{3}{2}x}$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
	M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Mark scheme and Additional Guidance continues on the next page

<table>
<tr><td rowspan="10">22 cont</td><td colspan="4">Alternative method 4 Takes the xth root of each side and uses powers of 2</td></tr>
<tr><td>$(16^x =)\ 2^{4x}$
or
$16^x = \left(\frac{1}{2^{3x}}\right)^{\frac{1}{x}}$</td><td>M1</td><td colspan="2">oe eg $16^x = \sqrt[x]{\frac{1}{2^{3x}}}$ or $16^x = \frac{1}{2^3}$
or $16^x = 2^{-3}$
may be implied by 3rd M1</td></tr>
<tr><td>$2^{4x} = \left(\frac{1}{2^{3x}}\right)^{\frac{1}{x}}$</td><td>M1dep</td><td colspan="2">oe eg $2^{4x} = \frac{1}{2^3}$
may be implied by 3rd M1</td></tr>
<tr><td>Correct quadratic equation
or correct linear equation
or correct equation involving indices with the same base</td><td>M1dep</td><td colspan="2">eg $4x = -3$ or $2^{4x} = 2^{-3}$
do not allow if the equation is from incorrect working
do not allow if the only equation is $x = -\frac{3}{4}$</td></tr>
<tr><td>M3 and $-\frac{3}{4}$</td><td>A1</td><td colspan="2">oe
ignore inclusion of answer 0</td></tr>
<tr><td colspan="4">Additional Guidance</td></tr>
<tr><td colspan="3">Up to M2 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td colspan="3">Allow $2^{4 \times x \times x}$ for 2^{4x^2} etc</td><td></td></tr>
<tr><td colspan="3">Responses using other powers eg powers of 8 can be escalated</td><td>Escalate</td></tr>
<tr><td colspan="3">Ignore simplification or conversion if correct answer seen</td><td></td></tr>
</table>